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SELF-ADAPTIVE CONTROL SYSTEMS.

UNITED STATES
NAVAL POSTGRADUATE SCHOOL

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SUPPORTED IN PART BY THE OFFICE OF NAVAL RESEARCH
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TABLE OF CONTENTS

Section		Page
1.	Statement of the Problem	1
2.	Performance Criteria	4
3.	Discontinuous Feedback Compensation	6
4.	Discontinuous Cascade Compensation	8
5.	Conditional Feedback	9
6.	Root Location Control	13
7.	Evaluation of the Impulse Response and Possible Uses in Self Adaptive Problems	16
8.	Self Adaptation of the Lateral Response of a Supersonic Aircraft	18
9.	Areas for Advanced Study in the Self Adaptive Control of Aircraft	22
	Tables	26
	References	31

ABSTRACT

The control of supersonic aircraft presents a difficult problem because the variation in aerodynamic parameters over the range of flight conditions encountered is too great to be compensated for by conventional techniques. It is generally recognized that the problem is basically nonlinear in nature, and the usual procedure is to use some type of nonlinear compensation to permit self adaptation within the system. Most of the schemes which have been proposed to date are reviewed here in considerable detail, and it is seen that the mechanization required is rather complex. The basis for a proposed study is then developed. In essence it consists of the use of active networks in the autopilot to provide complex zero compensators which confine the excursions of roots during parameter variation to acceptable areas on the s-plane. The proposed mechanization is simple, and it may be possible to reduce self adaptation to the status of a vernier adjustment, or eliminate it entirely.

1. STATEMENT OF THE PROBLEM

Linear servo theory is concerned primarily with systems described by differential equations with constant coefficients. Time varying coefficients may also be handled (though not as readily) and sampled data systems may also be considered linear. Nonlinear servo theory, on the other hand, is concerned primarily with systems for which the coefficients of the describing differential equations vary as functions of some dependent variables. Usually the coefficients are functions of signal amplitude, or of a frequency of oscillation, or of a velocity or acceleration. For such systems the nonlinearity can be determined by measurement. Thus it is known quantitatively and its characteristics may usually be considered invariant with time and conditions external to the system. Another class of nonlinear systems, however, are those for which parameter values change during operation as functions of some independent variable. For example, the parameter of a continuous chemical process may change as functions of ambient temperature; as functions of the rates of inflow and outflow, etc.; in a paper mill the inertia of the winding reel changes as the paper is wound on, and the motor torque required to maintain constant tension in the web changes with reel diameter; in a missile the mass and the center of gravity change as the fuel is consumed; in a supersonic aircraft the aerodynamic coefficients change with altitude. In general the basic differential equations may be either linear or nonlinear, depending on the physical nature of the process, the variation in parameters due to independent variables is an additional effect. For linear systems, and for the usual nonlinear system a fixed compensator is adequate providing it can be devised. When independent parameter variation is encountered, however, a fixed compensator

may be adequate only over a restricted range of operating conditions. Outside of this range the compensation must be changed, i.e., the system must be adapted to the new conditions. When the system is designed to automatically change its own compensation, then it may be called SELF ADAPTIVE.

The solid line block in Fig. 1 represents a variable parameter process, and the dotted lines represent additions that might be used to incorporate the process into a feedback control system. In general the basic physical nature of the process is known, describing differential equations or transfer functions can be written in symbolic notation, and usually parameter values can be supplied for at least one set of operating conditions. The variation in parameters is seldom well defined. In some cases the variation is known to be small and upper and lower limits for the parameter values may be available. For other cases the range of variation is known to be wide and for extreme values are known to make the process unstable. It is always possible to measure the effects of parameter variations by methods which determine the impulse response of the system (see section 7), but such tests may be prohibitively expensive both economically and in terms of time lost. In any event, the use of feedback control techniques is expected to provide an overall system which is stable and performs satisfactorily under all operating conditions.

When a component in a feedback control system (such as the process block in Fig. 1) causes unsatisfactory performance such that compensation is required the function of the compensator may be studied from several viewpoints. Fig. 1 shows blocks for a cascade compensator and a feedback compensator. One point of view is that the function of these compensators is to alter the open loop transfer function. The desired result may

be specified in terms of frequency response characteristics or root locus characteristics (these are equivalent for linear processes). Compensation designed on this basis may be adequate for processes with a small range of parameter variation even though the compensator components are fixed elements. It seems possible that special fixed element compensators may be satisfactory in some cases of wide variations in process parameters (See section 9). In general, however, it seems probable that the fixed compensator cannot provide an acceptable transfer function over the entire range of process parameter variation. The obvious solution to this problem is to devise a means of adjusting the compensator parameters as the process changes its characteristics, thus providing an essentially invariant transfer function by a process of adaptation. Note that the concepts derived using this point of view are naturally analog type concepts, involving components that utilize continuous signals. The concept of discrete data and digital computing devices is not prohibited by the viewpoint, but would appear as an alternative rather than a first choice.

A second viewpoint which may be applied to the block diagram of Fig. 1 is that the desired output can be obtained from the variable parameter process by shaping the signal input to the process block. Using this approach the blocks designated as compensators are considered to be computing devices which accept measured data from the system input and output, operating on this data to compute the proper signal for application to the process block. The computer scheme may be an analog device, perhaps with servo loops to permit parameter variation (in which case the net result may be the same as that obtained using the compensator viewpoint) or it may be a digital computer with an analog converter to provide the proper physical form of signal

to the process. In either case the computer must be programmed, i.e., it must be given instructions as to how the measured data should be operated on to determine the proper input signal. This simply means that some performance criterion must be built into the system.

2. PERFORMANCE CRITERIA

In the literature of adaptive control systems the word "optimum" is commonly used to designate the performance which the adaptive scheme is attempting to achieve. In relay servo theory the word "optimum" designates dead beat response to a step input utilizing maximum effort drive at all times. In adaptive applications the word seldom if ever has this meaning, nor is there a single meaning for the word. Some performance criteria are based on the desired response to a step input. For example, in the adaptive control of aircraft, studies at the Cornell Aeronautical Laboratory[1,2] have shown a human pilot preference for a response which is typical of a second order system with $\zeta = 0.7$ and $\omega_n = 3.0$. Then an optimum performance criterion is that the system response to all commands and to all disturbances should be the same as the response that would be obtained if the system were second order with $\zeta = 0.7$ and $\omega_n = 3.0$. Slight extensions of this reasoning easily lead to criteria which retain the second order system concept, but specify a fixed ζ for all conditions, permitting reasonable variation in ω_n, or conversely a fixed ω_n may be required with reasonable variation permitted in the value of ζ. The differences in the definitions naturally lead to different requirements as to measuring schemes, and consequently different compensators are evolved. When both ζ and ω_n are specified it is convenient to use a model to specify the required performance. Fig. 2 shows two techniques

which have been applied to use models in adaptive systems. In Fig. 2a the adaptive scheme is independent of the model and is included in the compensated process. The model itself is placed between the command signal and the closed loop system, thus shaping the command signal to a form which is actually the desired response. This is applied to the closed loop system, and since the desired response signal obviously varies more slowly than the actual command, a tight loop can follow this signal so that the error is always small. Fig. 2b shows the use of a model in a feedforward path. This scheme has been called a conditional feedback system,[3] the feedback of a control signal being conditional on the result of a comparison between the model output and the measured output characteristics. The model is connected in a feedforward path, and the system operates open loop except when a corrective feedback signal is necessary. Correction for parameter variation is obtained by making the feedback path adaptive.

Note that these model schemes are instantaneous schemes since all signals are presumably continuous. Thus there is an attempt to minimize the instantaneous error. When the model is not used an instantaneous reference value may not be available, as in cases where ζ is to be kept constant, or ω_n is to be kept constant. In such cases the error signal may be defined by comparison of the output with the actual command or disturbance signal. To optimize performance under these conditions the criterion might be to minimize the average of the squared error, or the integral of the magnitude of the error evaluated during a finite period, or some similar scheme. For such performance criteria the computing scheme must include the proper components to evaluate the quantity chosen, and this might be called a model. The mechanization of any such scheme may utilize either analog or digital means, and

for certain types of criteria the mathematical computations required make digital schemes very attractive.

3. DISCONTINUOUS FEEDBACK COMPENSATION

A study by Flugge-Lotz and Taylor[4] has shown that self-adaptive action can be obtained by utilizing nonlinear feedback paths for compensation. The basic proposal is to feedback the output and the first derivative of the output around the system, making the gains of both paths variable. The gains are variable in steps rather than continuously, are phase (or polarity) adjustable, and the values in use at any instant are selected automatically by a predetermined switching logic.

Consider the block diagram of Fig. 3. The basic system is assumed to be linear and second order (a convenient, but not a necessary assumption). There is a velocity feedback path with transfer function $k_0 s$, and a direct feedback path with transfer function k_1. Note that for a given switch setting k_0 and k_1 are constants, but each has four possible values. The system function for the compensated system is readily derived. The forward transfer function is

$$G_1 = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{1}$$

and the transfer function of the two parallel feedback paths is

$$G_2 = k_1 + k_0 s \tag{2}$$

Thus the system function is

$$\begin{aligned} \frac{\theta_c}{\theta_R} &= \frac{\omega_n^2/(s^2+2\zeta\omega_n s + \omega_n^2)}{1 + \omega_n^2(k_1+k_0 s)/(s^2+2\zeta\omega_n s+\omega_n^2)} \\ &= \frac{\omega_n^2}{s^2 + (2\zeta\omega_n + \omega_n^2 k_0)s + \omega_n^2(1+k_1)} \end{aligned} \tag{3}$$

In differential equation form this becomes:

$$\ddot{\theta}_c + (2\zeta\omega_n + \omega_n^2 k_0)\dot{\theta}_c + (1+k_1)\omega_n^2\theta_c = \omega_n^2\theta_R \tag{4}$$

Since k_0 and k_1 are variables (in a discontinuous sense) the differential equation is nonlinear, and the damping and natural frequency may be considered variable quantities. If the basic system is linear (ζ and ω_n constant) but θ_R has a regular (or random) variation with time, then the changes in k_0 and k_1 may be thought of as parameter adjustments which attempt to minimize the instantaneous error. If θ_R is relatively constant, but the basic system is nonlinear in the sense that ζ and ω_n change with operating conditions, then the changes in k_0 and k_1 may be considered a self adapting process which attempts to keep the equivalent damping and natural frequency invariant.

Derivation of the switching logic is not attempted here but is available.[5] The result is

$$a_m = -A_1 \frac{\dot{\theta}_c E}{|\dot{\theta}_c E|} - A_2 \frac{\dot{\theta}_c \dot{E}}{|\dot{\theta}_c \dot{E}|} \tag{5}$$

$$b_m = -B_1 \frac{\theta_c E}{|\theta_c E|} - B_2 \frac{\theta_c \dot{E}}{|\theta_c \dot{E}|} \tag{6}$$

$m = 0, 1, 2, 3$

A_1, A_2, B_1, B_2 = positive constants

$$\begin{aligned}
a_0 &= -A_1 - A_2 & \qquad b_0 &= -B_1 - B_2 \\
a_1 &= -A_1 + A_2 & \qquad b_1 &= -B_1 + B_2 \\
a_2 &= A_1 - A_2 & \qquad b_2 &= B_1 - B_2 \\
a_3 &= A_1 + A_2 & \qquad b_3 &= B_1 + B_2
\end{aligned} \tag{7}$$

From equations 5 and 6 it may be noted that switching occurs whenever any one of the control variables (θ_c, $\dot{\theta}_c$, E, $\dot{E}$) changes sign. Because of this equations 5 and 6, when properly manipulated,[4] lead to a digital computer technique for mechanizing the switching logic.

The selection of values for the feedback gains a_0, a_1, b_0, b_1, etc., is a matter of experimental trial and error, and depends on the normal characteristics of the process and the expected range of parameter variation within the process, as well as the expected variation in the input signal.

Preliminary studies[4] have extended this philosophy to the control of a third order linear process. A single such system was studied, using position and velocity feedback in the same scheme as for the second order system. The switching logic was the same as for the second order system, but the feedback gains a_0, a_1, b_0, b_1, etc., were adjusted to fit the third order system. Performance was much improved, proving that the scheme can work for higher order systems.

4. DISCONTINUOUS CASCADE COMPENSATION

The need for self adaptive control has been most apparent in supersonic aircraft. In this area the use of discontinuous feedback compensation does not appear attractive because of the complexity of the switching logic. Early[6] studies of the control of aircraft or missile pitching motion indicated that relay control with linear derivative switching provided good adaptive performance. The basic block diagram is shown on Fig. 4. In essence this is merely a relay servo, and may be analyzed and synthesized by application of describing function methods. Further investigation[7] extended the study to pitch rate control of a simulated supersonic fighter. Initial studies utilized the configuration of Fig. 5. Note that the anticipatory switching is quite elementary. Optimum relay servo theory was not attempted because mechanization of the switching requirements is prohibitively complex for the high order system. Computer tests were satisfactory, and the method was applied to a more realistic model as shown in Fig. 6.

The loop of Fig. 5 is readily analyzed using describing function methods. In general the results show a very tight loop, but a limit cycle is unavoidable when an ideal relay is used. However, the insertion of a lead network between the error detector and the relay decreases the amplitude of the limit cycle, and use of a relay with dead zone permits elimination of the limit cycle. When an ideal relay is to be used, its characteristics can be altered by superimposing a dither signal. Relay characteristics as affected by a dither signal are shown in Fig. 7.

The inner loop, when stabilized in this adaptive fashion, is so tight that the response to abrupt commands[8] would be considered too harsh by the pilot. Therefore input signals are shaped by a model, the model used being quadratic[9] and using $\zeta = 0.7$, $\omega_n = 3.0$. The technique was applied successfully to a Lockheed F-94C. The block diagram is shown in Fig. 8. Two additional features are noteworthy, the gain changer-limiter combination, and the filter following the limiter. The gain changer-limiter permits full corrective action for large errors, but restricts the corrective action for small errors. This permits rapid corrections but gives greater stability at steady state. The filter has two purposes; ideally the denominator cancels the numerator of the servo and actuator, while the numerator of the filter is intended to compensate for the backlash in the aircraft control system.

5. CONDITIONAL FEEDBACK[3]

Consider the block diagram of Fig. 8. The command signal θ_R is fed into two transfer blocks A and B. The signal transmitted through A may be considered as an actuating signal, causing an output θ_c. On the other hand the second channel through block B creates a signal, a, at the

output of block B, and this signal is compared with a feedback signal, b, which is a function of the output signal θ_c. If $a \neq b$, a correction signal is transmitted through block H_2. Thus the existence of feedback from θ_c through H_1 and H_2 is conditional, depending on the inequality $a \neq b$.

By inspection of Fig. 8, if there is a command signal $\theta_R(s)$ and if

$$B(s) = A\ G_1\ H_1\ (s) \tag{8}$$

then

$$B(s)\ \theta_R(s) = A\ G_1\ H_1\ (s)\ \theta_R(s) \tag{9}$$

and

$$a \equiv b \tag{10}$$

Thus it is possible to build a system so that the forward transfer function $AG_1(s)$ is exactly the desired relationship between $\theta_c(s)$ and $\theta_R(s)$. The system effectively operates open loop for command signals, since no signal is transmitted through H_2, and thus the components B, H_1 and H_2 might as well be disconnected since they in no way affect the response to a command signal as long as equation 8 is satisfied.

If $H_1 \equiv 1.0$, then $B(s) = AG_1(s)$, which means that the transfer function of the block B should be identically the transfer function of the forward loop. Since this has been chosen to give exactly the desired output response on open loop operation, B is then a <u>model</u> for the system performance.

The feedback block H_2 can be designed to control the effect of disturbances. For an output disturbance D, as shown on Fig. 8, the output is

$$\theta_c(s) = g(s) + D(s) \tag{11}$$

but for the case of no command signal

$$g(s) = - H_1 H_2 G_1(s) \Theta_c(s) \tag{12}$$

and thus

$$\frac{\Theta_c(s)}{D(s)} = \frac{1}{1 + G_1 H_1 H_2(s)} \tag{13}$$

And it is readily seen that the feedback loop can be adjusted to satisfy disturbance-response specifications by proper design of H_2.

Using the superposition principle the response of the system to simultaneous command and disturbance signals is

$$\Theta_c(s) = \frac{D(s)}{1 + G_1 H_1 H_2(s)} + AG_1(s)\Theta_R(s) \tag{14}$$

Thus the response to a command is independent of the response to a disturbance. It is at this point that the possibility of self-adaptive operation becomes apparent. The disturbance term in equation 14 can be made quite small by proper design, can be made zero in steady state and perhaps can be made to approach zero during transient operation for certain interpretations of the meaning of the disturbance signal, D. In terms of the block diagram of Fig. 8, there is signal flow through element H_2 because $a \neq b$, i.e., the signal fed back from Θ_c does not match the output of the model. The system does not distinguish between signal differences due to an actual output disturbance, and signal difference due to parameter changes in G_1. Thus the signal D may represent either, and in either case the feedback loop tries to minimize the effect of the change. This is a type of self adapting procedure. As applied to the problem of self adaptation in aircraft, consider the block diagram of Fig. 9. Here the model is designed to have the desired second order characteristics ($\zeta = 0.7$; $\omega_n = 3.0$) although the combined aircraft-actuator system may have a somewhat different transfer function. If the aircraft-compensator loop is designed to be well damped

the model operates on command inputs to provide the desired response times to commands. It might be said that the feedback configuration should be selected to force the objectionable airframe roots into a region of high damping, then the model provides the dominant roots which control system performance as long as the airframe roots are not permitted to move into this region.

When operating with supersonic aircraft the airframe roots may move appreciably due to the variation in aerodynamic coefficients. The feedback tends to minimize the effects of these variations. Essentially the model provides positive feedback to speed up the response when the aircraft is responding too slowly, and provides negative feedback to slow down the response when the aircraft is responding too rapidly. In order to be effective over a wide range of parameter variations, however, the feedback gain must be kept high. This is not always feasible practically, and limitations in the feedback gain permit the variations in the airframe roots to have some affect on the transient response of the system.

A modification which permits the scheme to be more truly adaptive is indicated in Fig. 10. The scheme presented is the addition of a compensator in cascade with G_1, and a subordinate servo unit driven by the difference signal from the model-output comparator. When parameter changes occur in the process (G_1) the difference signal drives the servo which adjusts a parameter in the compensator, thus effectively cancelling the original parameter variation. Note that the compensator in Fig. 10 is in cascade with G_1, but this is only one possibility, a feedback or a feedforward compensator might be more practical in a given case.

When parameter adjustment is used as a self adaptive mechanism the designer naturally attempts to find the simplest and most practical adjustment

available. In most cases this reduces to a gain adjustment, often in a feedback path. Adjustment of a circuit time constant is sometimes the best solution, and the discontinuous feedback scheme of Fig. 3 might be modified to include conditional feedback in place of the switching logic.

When the process to be controlled is subject to parameter variation, and the self adaptive scheme used includes a minor servo loop which adjusts a gain (or other parameter) then some provision is needed to assure continual self adjustment. The problem is this; the command signal and output signal may both be set at some constant value and remain unaltered while environmental conditions cause appreciable change in the process parameter values. With no change in either system output or model output the adjusting servo does not operate and the compensator is not altered to adapt the system. If a command is given or a disturbance is encountered the system responds, and the subordinate servo loop attempts to perform its adaptive function in spite of initial misalignment. Such conditions can lead to limit cycles or even complete instability. To prevent this some scheme to maintain continual adjustment of the adaptive compensator is desired. One technique is to pulse the system with a low amplitude pulse or impulse at some regular (or random) repetition rate. The response thus excited can usually be kept below the perception level of human operators, but is still large enough to force the adaptive system to maintain a proper adjustment. In some applications it is thought that the inherent noise level may be sufficient to activate the adaptive loop.

6. ROOT LOCATION CONTROL[8]

In any automatic flight control system the complexity of the structure and control loops provides a differential equation of high order. The roots

of the characteristic equation are usually all complex conjugate roots as shown[10] in Fig. 11. These roots are separated by a factor of 5 to 10, thus the loops represented by the roots are not tightly coupled and each loop can usually be stabilized during design without seriously altering the characteristics of any other loop. However, in the normal design of the loops which stabilize the airframe dynamics, gain control has the usual effect, i.e., increasing the gain drives the roots toward the right half of the s-plane. This, of course, seriously restricts performance.

If attention is restricted to the problem of stabilizing the airframe, and if the measuring instruments can be mounted at airframe nodes so that the bending modes are not coupled into the control system, then only the phugoid, short period and servo loop roots need be considered. By proper design of compensation zeros may be introduced into the loop transfer function in such a way that the roots move toward the negative real axis as the gain is increased. This is shown qualitatively on Fig. 12. With this type of design the servo loop roots still move toward the right half plane as the gain is increased, but the gain required to make the system unstable is considerably greater than would be the case when the phugoid or short period roots move toward the right half plane. Thus the gain can be raised until the airframe roots are all real, and an overdamped, sluggish, but very stable system results.

In order to utilize maximum permissible gain while maintaining stability as parameters vary, it is noted that the frequency of the servo loop roots increases as they move toward the right half plane. The effect of parameter variation in the airframe equations is to alter the location of the phugoid and short period roots, and this effect acts on the servo loop in a fashion

similar to a gain change, causing the servo loop roots to move along a locus similar to the root locus of Fig. 12. They may be returned to their desired location by changing the gain of the servo loop. An error detection scheme which permits mechanization of this gain adjustment is shown on Fig. 13. The technique is to pulse the system periodically, monitoring the output of the servo actuator. The frequency of this output is measured by using the oscillations to form a pulse train of constant amplitude pulses, which are then counted to determine the frequency. If the measured frequency is other than the preselected value the gain is adjusted to return the servo loop roots to the proper location. Note that the underdamped servo loop roots are not dominant, they do not even affect the transient performance of the system noticeably as long as they are forced to remain at or near their preselected location. If they are permitted to move toward the imaginary axis, however, their contribution to the transient response becomes appreciable, and instability results if the roots cross into the right half plane. The use of the gain adjuster is then a means of assuring use of maximum gain consistent with the requirement that the system remain stable with good transient performance; the frequency of the servo loop transient is just a convenient reference for use in making the gain adjustment. Note also that the actual transient response is dominated by the roots of the compensated airframe, and is thus an overdamped response, so that system (i.e., aircraft) performance is sluggish, which is undesirable.

In order to provide acceptable aircraft performance when the overdamped self adaptive aerodynamic system is used, the actuating signals may be shaped by a nonlinear input network as shown on Fig. 14. This network creates an artificial signal which forces the aircraft to respond to a command or

disturbance according to pilot preferences, i.e., the model used shapes the actuating signal to compensate for the overdamping which is characteristic of the real roots. Note that this arrangement, while unusual, is fail safe in the sense that it may be disconnected and the aircraft is still very stable, though sluggish.

7. EVALUATION OF THE IMPULSE RESPONSE AND POSSIBLE USES IN SELF ADAPTIVE PROBLEMS

If the process to be controlled has variable parameters, but these parameters can be considered constant during some finite time interval, then the process may be considered linear during this time interval (assuming no inherent nonlinear components). During this time interval the process may be said to possess an impulse function, or weighting function $g(t)$. This weighting function must change as the parameters change, but if it can be evaluated for a given interval, then the process characteristics are determined for that interval and this information can be used to control the process.

Assuming that a given process can be represented by some weighting function $g(t)$, then for any input $e_i(t)$ the output $e_o(t)$ is given by the convolution integral

$$e_o(t) = \int_{-\infty}^{\infty} e_i(T) g(t - T) dT \tag{15}$$

Multiplying both sides by $e_i(t - T_1)$

$$e_o(t)\, e_i(t - T_1) = \int_{-\infty}^{\infty} e_1(T)\, e_i(t - T_1) g(t - T) dT \tag{16}$$

The cross correlation function of the input and output is defined to be, at time T_1:

$$\phi_{io}(T_1) = \overline{e_i(t - T_1)\, e_o(t)} \tag{17}$$

where the bar indicates a time average. Thus, from equation 16

$$\phi_{io}(T_1) = \int_{-\infty}^{\infty} \overline{e_i(T)\, e_i(t - T_1)}\; g\,(t - T)dT \tag{18}$$

and from this

$$\phi_{ii}\,(t - T_1 - T) \stackrel{\Delta}{=} \overline{e_i(T)e_i(t - T_1)} \tag{19}$$

is the autocorrelation function of the input. Therefore

$$\phi_{io}(T_1) = \int_{-\infty}^{\infty} \phi_{ii}(t - T_1 - T)\; g(t - T)dT \tag{20}$$

The relationship of equation 20 may be used to determine the impulse function $g(t)$. If $e_i(t)$ is white noise of unit spectral density, then

$$\phi_{ii}(t - T_1 - T) \stackrel{\Delta}{=} \delta\,(t - T_1 - T) \tag{21}$$

where $\delta(t)$ is the unit impulse. Substituting in equation 20

$$\phi_{io}(T_1) = \int_{-\infty}^{\infty} (t - T_1 - T)g(t - T)dT \tag{22}$$

from which

$$g(T_1) = \phi_{io}(T_1) \tag{23}$$

That is, the cross correlation function of the output and the input delayed T_1 seconds is equal to the impulse response evaluated at $t = T_1$. If enough points are evaluated the impulse curve is determined. A number of techniques may be used to evaluate the cross correlation function. In general noise must be fed into the process and into a computer of some sort, and the process output must also be fed into the computer. The computer must then form the product $e_0(t)\, e_i(t - T_1)$ and average this product. This is repeated for $t = T_2, T_3, T_4 \ldots T_n$, thus obtaining n points on the impulse response curve. Since the process presumably has variable parameters, the n points

must be evaluated and the impulse curve determined in a suitable short total time; then the entire computation is immediately repeated, etc.

There are a number of possible uses of such a technique. The most obvious is to use it for measuring purposes. If the process can be made to operate over the entire range of expected operating conditions while the impulse response is being measured repeatedly, the results of these measurements completely define the range of variation in the effects caused by parameter changes. With this information available the engineer may be able to redesign the process, or may be able to apply relatively standard design techniques to the development of a control system for the process.

On the other hand, this method of measuring the impulse response $g(t)$ may be built into an adaptive controller for the process. One method is indicated in the block diagram of Fig. 15. The philosophy is based on the fact that knowledge of the command signal, the desired output signal in response to that command, and the impulse function of the process, the proper form of the actuating signal may be computed.

8. SELF ADAPTATION OF THE LATERAL RESPONSE OF A SUPERSONIC AIRCRAFT

This section is concerned with the details of one phase of a feasibility study. Adaptive control was felt to be required for a certain supersonic airframe. Transfer functions for six flight conditions were available from design data and preliminary tests. Several schemes had been suggested for mechanization of the adaptive control, and it was desired to analyze these so that an intelligent decision might be made for further investigation and development. It was necessary to check system response for oscillatory

stability over the entire range of flight conditions. Divergent instability due to a positive real root could be allowed if it was slight enough to permit the pilot or autopilot to remain on course by command inputs. It was desired that the system response have some similarity to a second order system so that attempts to approach the pilot preference condition of $\zeta = 0.7$ and $\omega_n = 3.0$ would be in order, as well as attempts to optimize using established second order principles.

The investigation was to be confined to lateral motion only. The analysis was to be linear with responses to be function of rudder deflection only. The aircraft was allowed three degrees of freedom as a result of these rudder deflections, and was assumed initially in steady level flight prior to application of the forcing function. For this discussion the output response is restricted to be the lateral acceleration. Symbols used are

N_y = lateral acceleration

δ_R = rudder deflection angle.

The block diagram of the loop is shown in Fig. 16.

The instrumentation in the control loop included an autopilot actuator which was a closed loop servo operating a hydraulic control value. The servo valve system had a transfer function

$$G_1(s) = \frac{800}{(s + 20 + j20)(s + 20 - j20)} \tag{24}$$

the main rudder actuator had a transfer function

$$G_2(s) = \frac{10}{s + 10} \tag{25}$$

and the accelerometer used to measure N_y had a transfer function

$$G_3(s) = \frac{15795}{(s + 87.9 + j89.7)(s + 87.9 - j89.7)} \tag{26}$$

The airframe transfer functions, N_y/δ_R, for six flight conditions ranging from landing conditions to high altitude, high dynamic pressure conditions are given in Table 1.

From Fig. 16 it is seen that the loop transfer function defining stability (and root location) consists of the product of equations 24, 25, 26 times the selected airframe transfer function from Table 1, times the adjusted gain K_{N_y}, times the transfer function of any cascaded compensator which might be used. For the case of no compensator the six loop transfer functions are as given in Table II. Root loci for these uncompensated loop transfer functions showed the basic system to be unsuitable for operation with negative feedback, and also with positive feedback. The root loci are not given here, except to note that in general the configuration is as shown on Fig. 17. It is interesting to note that the configuration for the supersonic aircraft differs from that of the conventional aircraft, which usually exhibits two complex pole pairs. For the supersonic aircraft the complex pole pair is the short period or dutch roll pair, and the loci emerging from this pair usually set the dominant, short period, high frequency oscillation. The real pair of poles occasionally give rise to a complex locus resulting in a pair of complex roots which provide the low frequency, long period oscillation commonly referred to as the phugoid mode. Characteristically the short period or dutch roll roots may attain reasonable damping, but the phugoid mode is usually very lightly damped. Real roots may also exist, usually one on the negative real axis and another near the origin but on either the positive or negative real axis. The former gives rise to the roll damping factor which controls the stiffness of response in roll to a rudder deflection. The root near the origin

determines the spiral characteristics of the aircraft, and as long as the residue is small the root may be permitted on the positive real axis because the pilot or autopilot can readily compensate the motion.

Several compensation schemes were investigated. Only one is considered here, a simple lead filter with transfer function

$$G_4(s) = \frac{s + 1}{0.1s + 1} \tag{27}$$

Using this filter the root loci for negative feedback exhibited undesirable characteristics, but if positive feedback is used the root loci give stable conditions over some variable gain range, and the loci of the short period roots indicated that the damping ratio can be controlled reasonably well by varying the loop gain. The root loci for all six flight conditions are given in Fig. 18. Gain points are marked on the complex loci. The numbers given are values of the variable gain K_{N_y}, not the total loop gain. Furthermore, the units of this gain are in deg/g.

By inspection of the root loci of Fig. 18, it was possible to select a root configuration for each flight condition which provided approximate second order system characteristics, with all stable roots and with reasonable damping for the dominant complex pair. The results of such a selection are given in Table 3. Note that an appreciable range of variation is required for the gain K_{N_y} if ζ and ω_n are to be kept nearly constant. Note also that ζ remains reasonably close to 0.3, and ω_n is nearly 2.0 for most flight conditions. This is not quite at the pilot preference value of $\zeta = 0.7$ and $\omega_n = 3.0$, but is close enough to warrant further investigation of adaptive means for adjusting the gain K_{N_y}. Further characteristics of the system which may be compiled from the root loci are given in Table 4. Note that no phugoid oscillations will occur, and only flight condition 3 has a spiral divergency condition. For flight conditions 1, 2, 5, 6 a higher frequency mode may go unstable if K_{N_y} is made too large.

The results of this analysis merely indicate that self-adaptive control by means of adjusting K_{N_y} is possible. One philosophy for the implementation of the control is indicated in Fig. 19. The fundamental concept is that of adjusting K_{N_y} to an optimum value. This might be accomplished by a servo driven potentiometer. It is only necessary to give the servo the proper instruction for adjusting the potentiometer. To accomplish this the airframe control loop might be pulsed periodically by a signal of sufficient amplitude to cause a measurable characteristic transient response, but of small enough amplitude to be below the perception threshold of the pilot. The output transient would be measured and supplied to a criterion computer whose function would be to minimize or maximize some preselected property of the transient response (such as adjusting to maximum $\int$). The signal from the criterion computer then commands the direction and magnitude of the servo drive in adjusting K_{N_y}.

Note that any such scheme is quite dependent on the nature of the criterion built into the computer. It is desirable that the criterion define a unique minimum (or maximum). It is also desirable that the minimum be reasonably sharp, yet the computation scheme must also be carefully chosen to prevent protracted hunting in approaching this minimum.

9. AREAS FOR ADVANCED STUDY IN THE SELF ADAPTIVE CONTROL OF AIRCRAFT

It is apparent that many specific schemes have been devised for the self adaptive control of aircraft, and that many more specific schemes may be devised. All of these schemes seem to be predicated on an a priori assumption of the feasibility of some type of adjustment; then mechanization schemes are developed which utilize this type of adjustment. A more fundamental approach to the problem would start with an analysis of the

effect of parameter variations in typical aircraft. Considerable information of this type is available,[12] but it has not been integrated into a systematic study of self adaptive control. It is also possible that additional information is needed. The first step should be the formulation of a block diagram for the controlled aircraft in three-dimensional space, including all cross-coupling blocks. Transfer functions should be obtained for all blocks with realistic numbers for gains and parameter values. Those parameters which are variable with flight conditions should be clearly marked and the functional (or other) relationship to flight condition established qualitatively and quantitatively.

The block diagram should then be manipulated into a standard form.[13] The block diagram should then be analyzed topologically and subdivided into sections referring to the airframe, control loops, etc. The smallest subdivision which contains all of the airframe should then be given a preliminary analysis. Established methods[13] permit ready evaluation of a characteristic determinant which is topologically related to the block diagram and which is readily used to undertake root locus studies.

The second step in the analysis should be a study of the effects of parameter variations. The procedure in this phase would be the establishment of the loop transfer function using the characteristic determinant and plotting the poles and zeros of this loop transfer function. The poles determined in this fashion are the poles of the actual hardware, the zeros are a result of mathematical manipulations. If plots are made for a number of flight conditions the poles and zeros will assume different locations for each flight condition. Superposition of the plots will establish s-plane trajectories for the poles and zeros, and these trajectories are an expression of the effect of parameter variation on the dynamic characteristics

of the system. Note that in the preparation of such plots the poles are established readily from the determinant but the zeros result from factoring a polynomial. This factoring can be expedited by use of an ESIAC computer.

Having established patterns for the effects of parameter variations on the loop transfer function pole-zero pattern several interesting questions arise as to the relationships between these variations and the aerodynamic coefficients and the relative importance thereof. At present these questions do not seem of sufficient importance to warrant further discussion. The next step would be the construction of root loci for each of a number of the basic pole-zero configurations. These are readily obtained with the ESIAC computer, and on each should be marked a number of points corresponding to the loop gain. Superposition of a number of these root loci will define areas on the s-plane in which the closed loop roots may lie. Loci of root movement for constant loop gain can be established, and if the loop gain variation with flight condition is noted, loci of root location as a function of flight condition can be established. Note that the term gain as used here refers to the loop gain defined by the determinant rather than the gain associated with any physical loop.

The existence of areas on the s-plane which define root locations is an unavoidable consequence of parameter variation. Compensation, in a broad sense, must mean the relocation and reshaping of these areas. We may then think of compensation as a two fold procedure; the definition of restricted regions in the s-plane, which may be defined as regions in which it is permissible to locate roots; then the design problem which is the problem of forcing the areas of actual root location to lie within the restricted regions. The concept of self adaptive control is based on the principle of

changing some system characteristic, usually a gain, in order to keep the closed loop roots within the restricted region. The usual theoretical treatment of self-adaptive control is developed in terms of keeping the dominant roots at a preselected point; the usual practical version is satisfied with some root movement. It is tacitly assumed in many self-adaptive schemes that fixed compensators cannot force the roots to remain in a sufficiently small restricted region on the s-plane. It has not been proven that this is true for most practical cases. For example the scheme of section 6 forces the dominant roots to remain on the real axis, which is considered to be a sufficiently restricted region, and adaptive control of the gain is needed only to prevent an instability of the servo loop roots. It may well be possible to design compensators which eliminate the need for any self adaptive sub-loop.

In the light of these comments, consider the fact that compensators can be designed which place complex zeros at arbitrarily designated points in the s-plane. One technique is the use of an active network of the type indicated by Fig. 20. Note that the transfer function of the device is

$$\frac{e_o}{e_i}(s) = K + K_1 \frac{(s + a)(s + b)}{(s + \alpha)(s + \beta)}$$

$$= \frac{K(s + \alpha)(s + \beta) + K_1 (s + a)(s + b)}{(s + \alpha)(s + \beta)} \qquad (28)$$

α and β provide real poles with essentially arbitrarily selected locations, and proper choice of K, K_1, a, b permits comples zeros of arbitrary location. Consider next that a combination of several such compensators with the basic pole-zero configuration of Fig. 17b permits a result as shown in Fig. 21. If this can be achieved the roots would be confined to a restricted region,

and self adaptive control may be unnecessary or in the nature of a fail-safe minor adjustment or trim.

To continue the basic study with these possibilities in mind, the results obtained from the study of the smallest subdivision of the block diagram which included the airframe should be expanded by adding additional segments of the block diagram as convenient. It is not anticipated that any major changes will be noticed in the root areas defined by parameter variation, nor in the general shape of the root loci but the expansion of the study to include the entire system should be made before investigating the possibilities of compensation. In general the studies should be carried out on the ESIAC, so the inclusion of additional remote poles and zeros is readily accomplished and a complete solution guaranteed without undue labor. Finally, any number of compensation schemes may be attempted, and with proper precautions the investigation may be accelerated by experimental methods, i.e., zeros and poles may be entered in the ESIAC and moved about to obtain a desired root locus and gain configuration. The mathematical and physical significance of the results may be verified later.

TABLE I

Three Degree Lateral Transfer Functions for N_y/δ_R at Indicated Conditions of Flight

Condition 1:

$$N_y/\delta_R = \frac{0.1479(s - 0.9468)(s - 0.0844)(s + 1.3209 \pm j\,0.5214)}{(s + 0.0657)(s + 0.7322)(s + 0.4715 \pm j\,1.5798)}$$

Condition 2:

$$N_y/\delta_R = \frac{0.5315(s - 0.0184)(s + 2.9354)(s - 2.1236)(s + 2.0986)}{(s + 0.0189)(s + 2.3472)(s + 0.3376 \pm j\,1.7172)}$$

Condition 3:

$$N_y/\delta_R = \frac{3.3485(s + 0.0063)(s - 4.1672)(s + 5.2181 \pm j\,0.4958)}{(s - 0.0007)(s + 5.209)(s + 0.532 \pm j\,3.4933)}$$

Condition 4:

$$N_y/\delta_R = \frac{0.2184(s - 0.0128)(s + 0.8625)(s - 1.2238)(s + 1.0598)}{(s + 0.0194)(s + 0.3334)(s + 0.187 \pm j\,1.6679)}$$

Condition 5:

$$N_y/\delta_R = \frac{3.3106(s - 0.0073)(s + 4.6913)(s - 5.9673)(s + 6.4883)}{(s + 0.044)(s + 4.7792)(s + 0.3571 \pm j\,2.3784)}$$

Condition 6:

$$N_y/\delta_R = \frac{0.8405(s - 0.001)(s + 0.9195)(s - 3.3749)(s + 3.489)}{(s + 0.0079)(s + 0.7404)(s + 0.1706 \pm j\,1.706)}$$

TABLE II

Open Loop Transfer Functions for the Lateral Response

N_y/δ_R at Indicated Conditions of Flight

Condition 1:

$$KG(s)_1 = \frac{0.1869\times10^8 K_{N_y}(s-0.9468)(s-0.0844)(s+1.3209\pm j0.5214)}{(s+0.0657)(s+0.7322)(s+0.4715\pm j1.5798)(s+10)(s+20\pm j20)(s+87.9\pm j89.7)}$$

Condition 2:

$$KG(s)_2 = \frac{0.6716\times10^8 K_{N_y}(s-0.0184)(s+2.9354)(s-2.1236)(s+2.0986)}{(s+0.0189)(s+2.3972)(s+0.3376\pm j1.7172)(s+10)(s+20\pm j20)(s+87.9\pm j89.7)}$$

Condition 3:

$$KG(s)_3 = \frac{4.2943\times10^8 K_{N_y}(s+0.0063)(s-4.1672)(s+5.2181\pm j0.4958)}{(s-0.0007)(s+5.209)(s+0.532\pm j3.4933)(s+10)(s+20\pm j20)(s+87.9\pm j89.7)}$$

Condition 4:

$$KG(s)_4 = \frac{0.276\times10^8 K_{N_y}(s-0.0128)(s+0.8625)(s-1.2238)(s+1.0598)}{(s+0.0194)(s+0.3334)(s+0.187\pm j1.6679)(s+10)(s+20\pm j20)(s+87.9\pm j89.7)}$$

Condition 5:

$$KG(s)_5 = \frac{4.1833\times10^8 K_{N_y}(s-0.0073)(s+4.6913)(s-5.9673)(s+6.4883)}{(s+0.044)(s+4.7792)(s+0.3571\pm j2.3784)(s+10)(s+20\pm j20)(s+87.9\pm j89.7)}$$

Condition 6:

$$KG(s)_6 = \frac{1.062\times10^8 K_{N_y}(s-0.001)(s+0.9195)(s-3.3749)(s+3.489)}{(s+0.0079)(s+0.7404)(s+0.1706\pm j1.706)(s+10)(s+20\pm j20)(s+87.9\pm j89.7)}$$

TABLE III

Root Locus Second Order Approximations of Transient Characteristics

N_y Plus Filter 1

Flight Cond.	K_{Ny} deg/g	ζ	ω_n rad/sec
1	20	0.312	1.8
2	5	0.45	2.2
3	0.6	0.22	4.6
4	0.7	0.29	3.8
5	20	0.25	2.0
6	5	0.42	2.4

Compiled Root Locus Characteristics for

N_y with Filter 1

Flight Cond.	Phugoid Characteristics	Short Period Characteristics	Real Root Characteristics	Unstable Mode	Remarks
1	None	Possibly at higher gains short period roots modified by H.F. locus at s=-10.0. Otherwise form dominant osc. pair	H.F. due loci from double pole	H.F. due loci from double pole	Short period locus entered neg. real axis, remained in stable plane
2	None	Formed dominant osc. pair. Possible modification by H. F. Locus	None in rt. half plane	H.F. due loci from double pole	Same as for flight cond. 1
3	None	Short period roots curve into rt. half plane. Dominance distorted by real root divergence.	System always contains unstable root	Divergent real root	Severe divergent instability prior short period pair entering rt. half plane. Locus from double pole remains in stable plane.
4	None	Dominant but curve into rt. half plane at increased gains.	None in rt. half plane	Short period pair	Same general pattern as cond. 3, however, no real root instability. Locus from double pole remains in stable plane, low and high osc. freq.

Flight Cond.	Phugoid Characteristics	Short Period Characteristics	Real Root Characteristics	Unstable Mode	Remarks
5	None	Possibly at higher gains short period pair modified by H.F. roots from double pole, other-wise dominant pair.	None in rt. half plane	H.F. due loci from double pole	Same as for cond. 1. The same pattern of loci existed as for cond. 1.
6	None	Same as for cond. 5	None in rt. half plane	H.F. locus from double pole	Same as for cond. 5.

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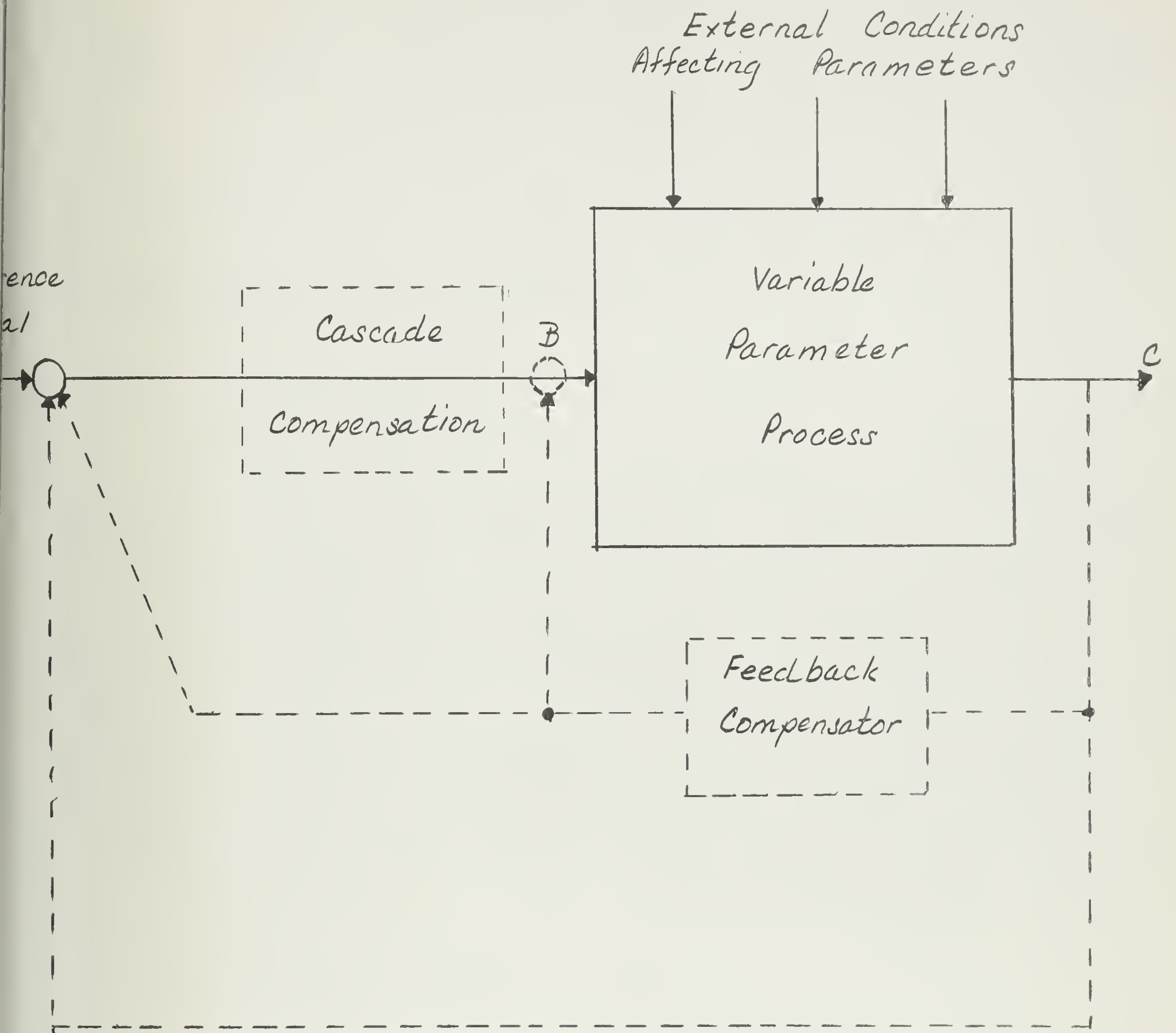
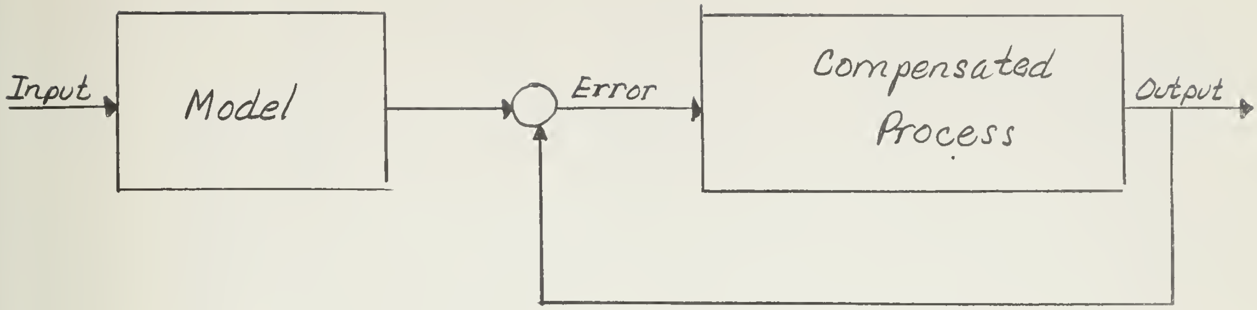
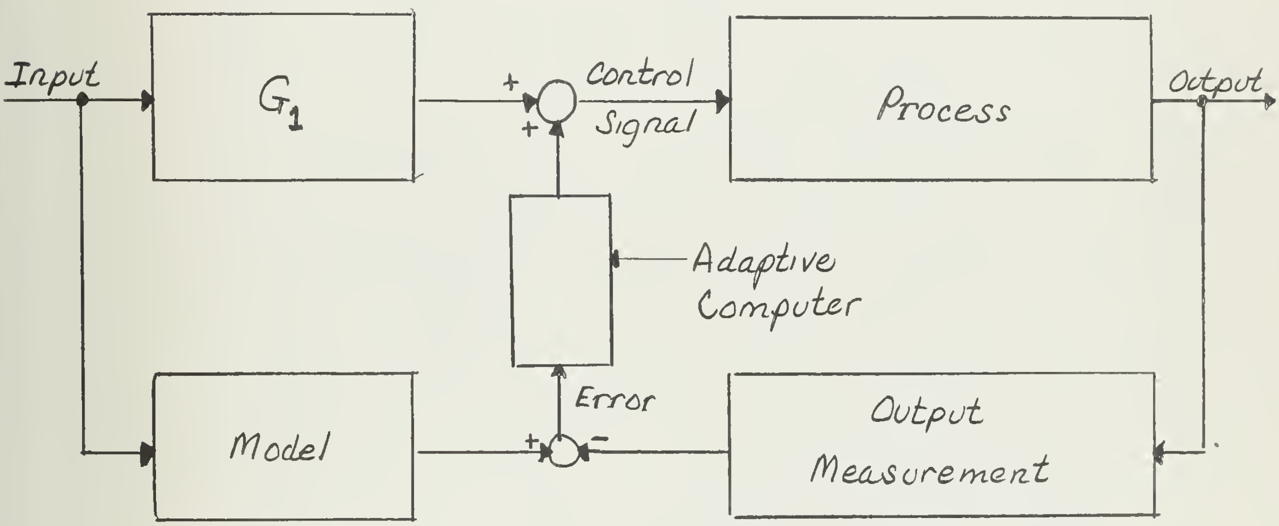


Fig. 1. Feedback Control of a Variable Parameter Process.



a) Use of a Model to shape Input Signal.



b) Use of a Model in Adaptation Process.

<u>Fig. 2. Use of a Model which defines the Performance Criterion.</u>

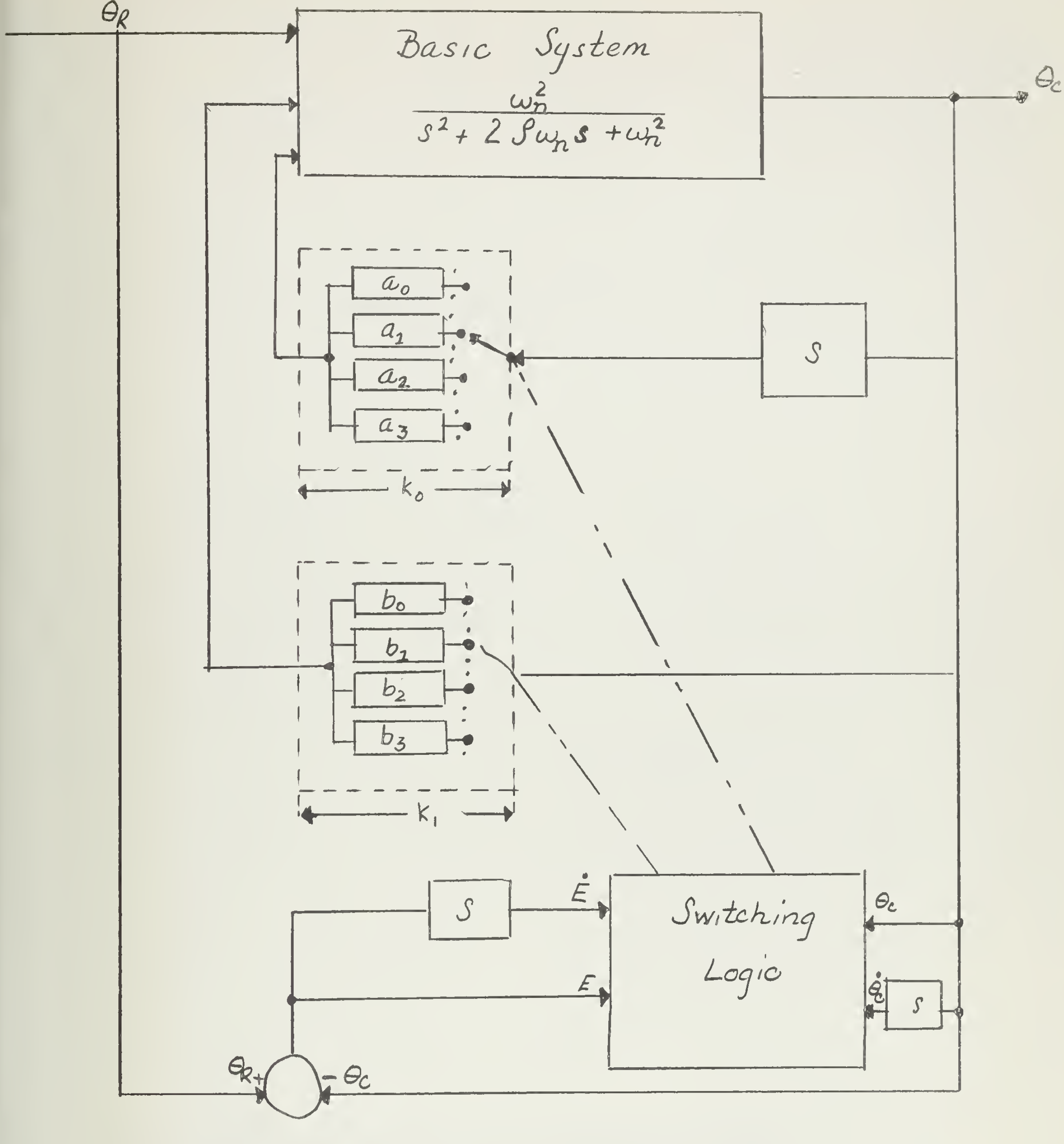


Fig. 3. Discontinuous Feedback Method for Self Adapting Systems.

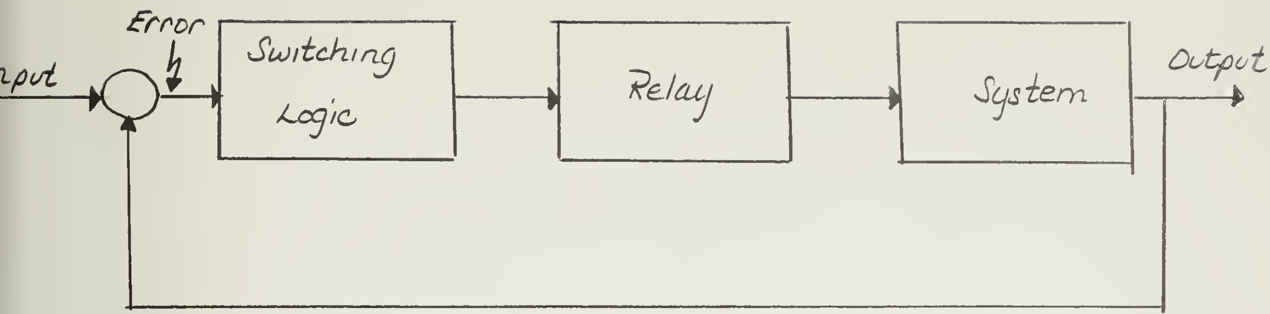


Fig. 4. Block Diagram for Cascade Discontinuous Control of a System.

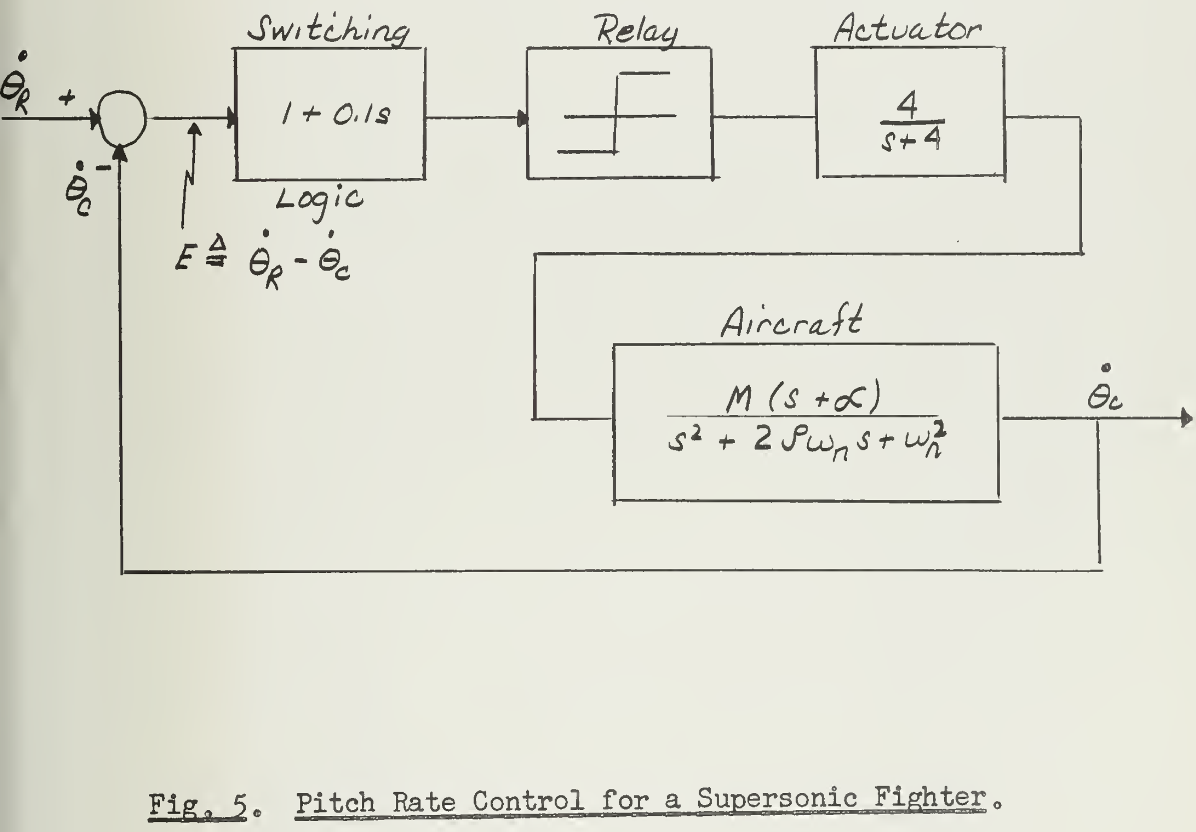


Fig. 5. Pitch Rate Control for a Supersonic Fighter.

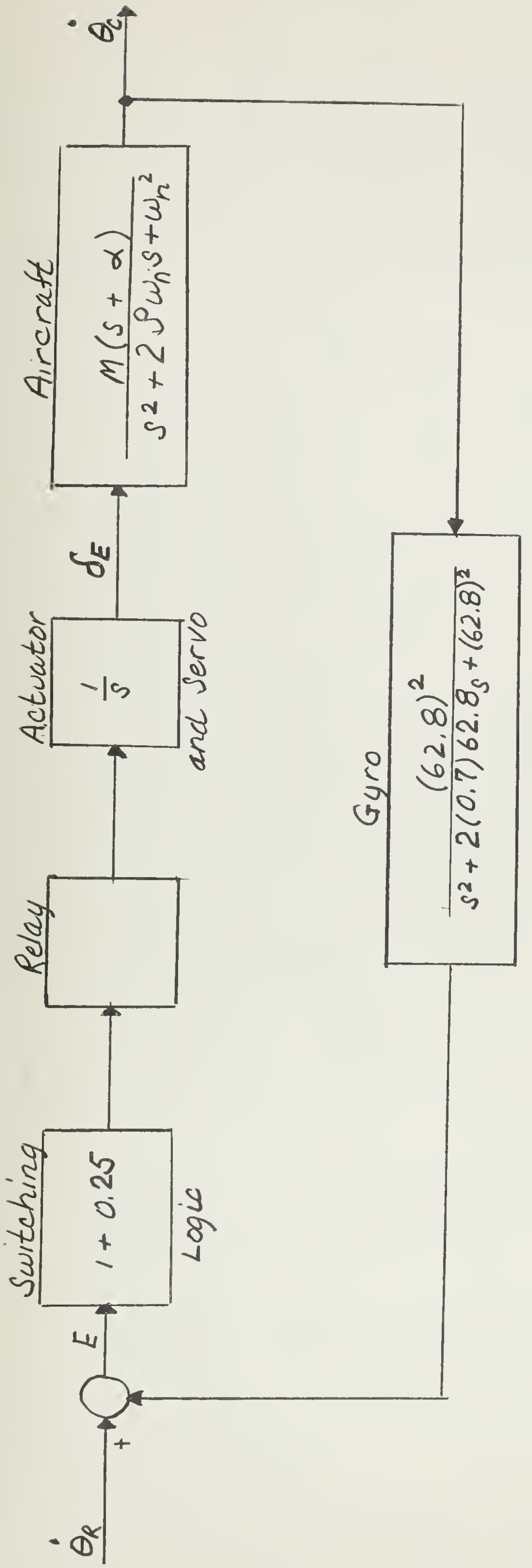
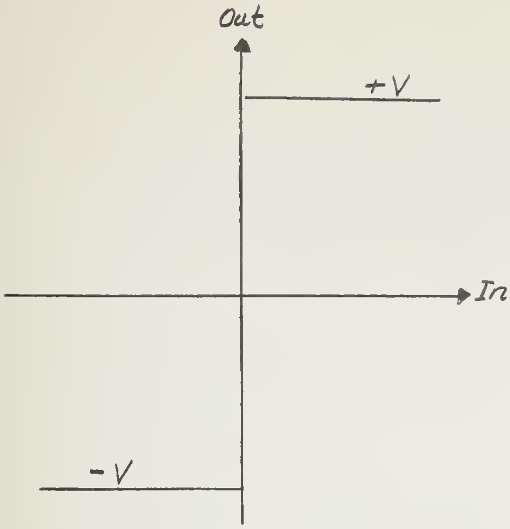
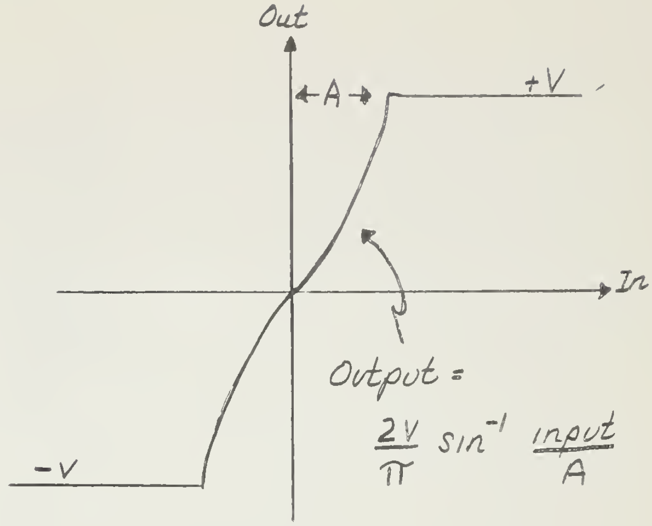


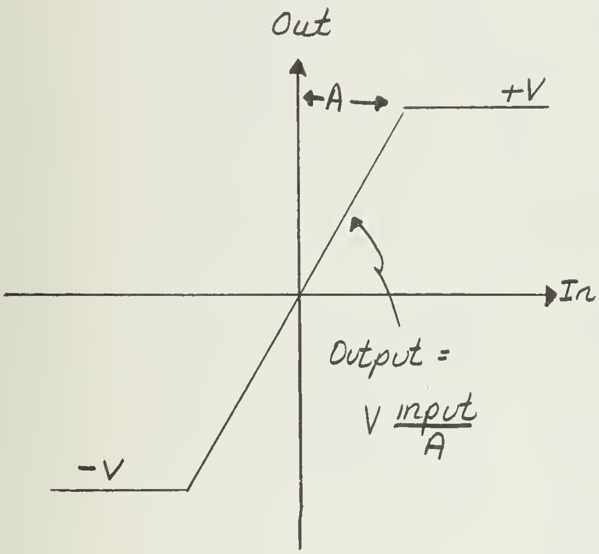
Fig. 6. Block Diagram for Pitch Control Loop Including Gyro Dynamics.



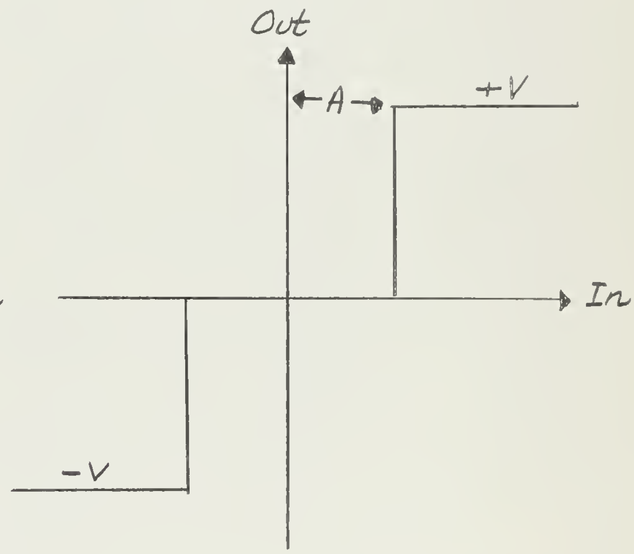
a) Ideal Relay



b) Ideal Relay with sine wave Dither = A sin ωt



c) Ideal Relay with Triangular wave Dither of Amplitude A



d) Ideal Relay with Square Wave Dither of Amplitude A

Fig. 7a. Effect of Dither on Relay Characteristics.

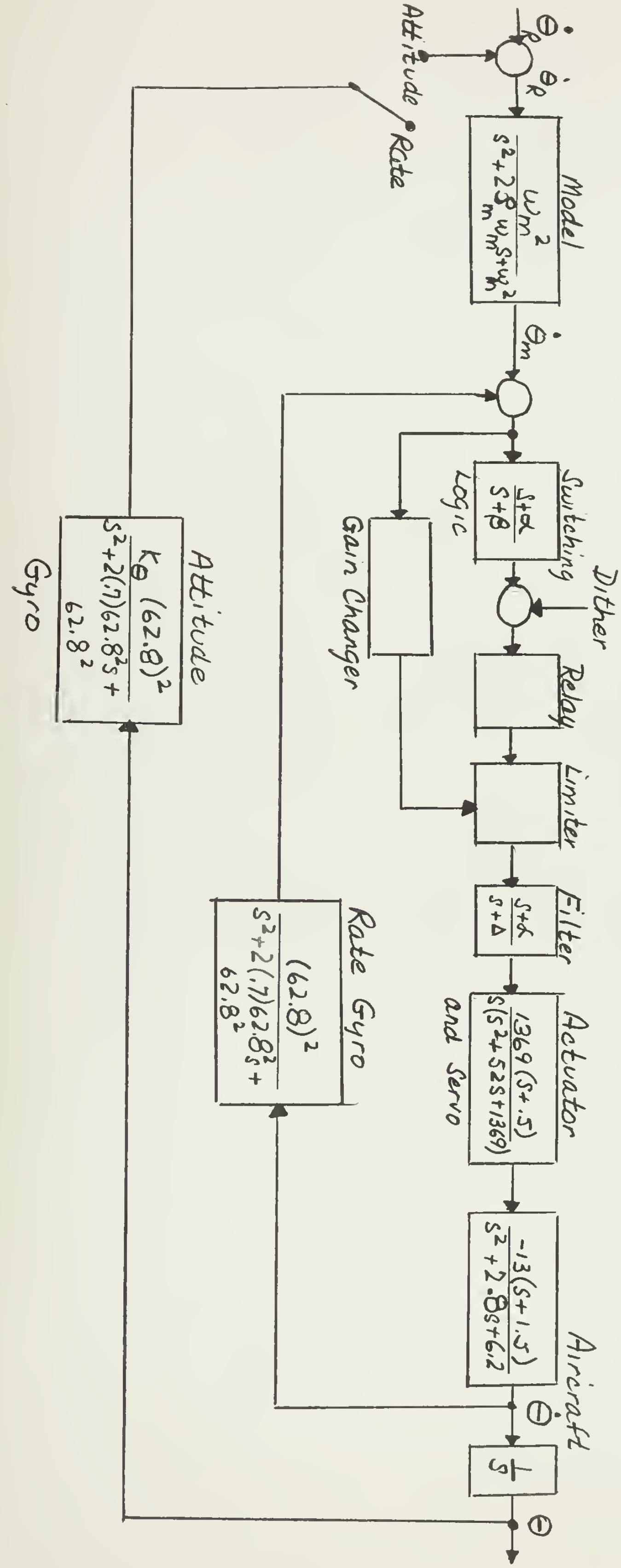


Fig. 7b. Adaptive Control System for F-94C (Numbers are for a Selected Flight Condition.

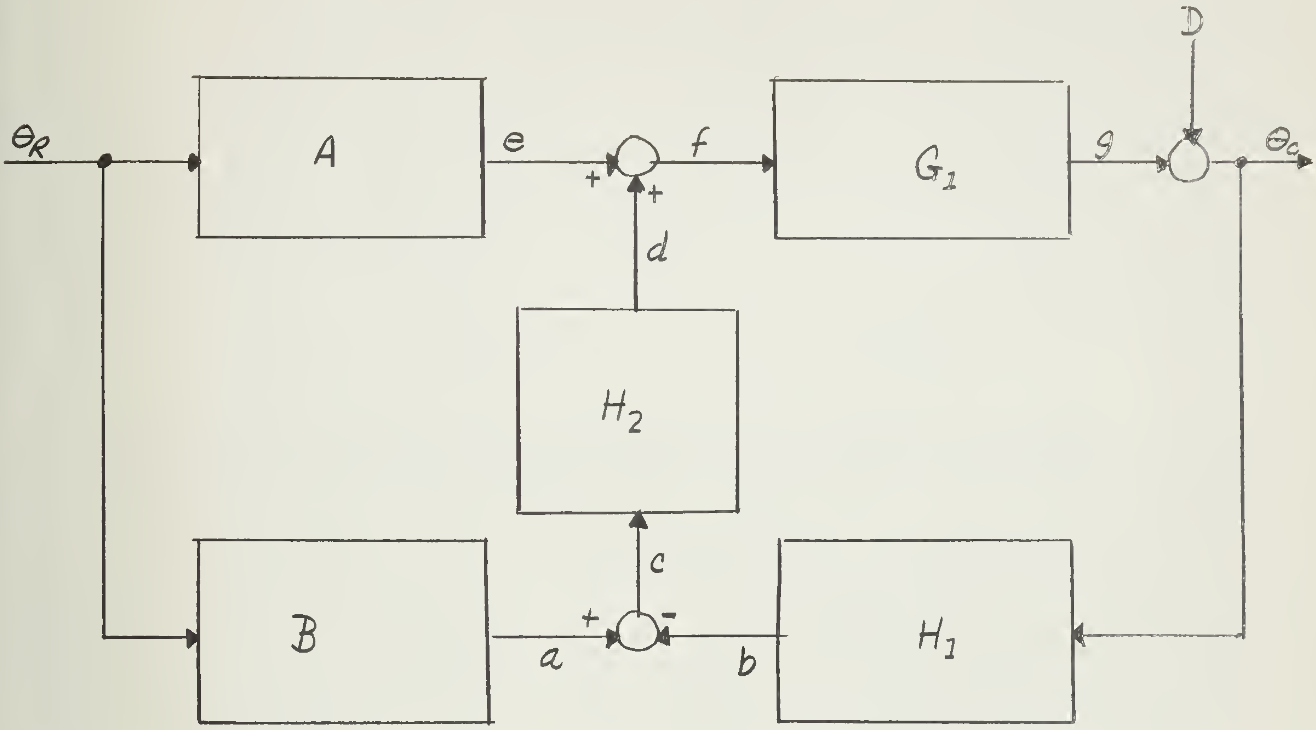


Fig. 8. Block Diagram for Conditional Feedback.

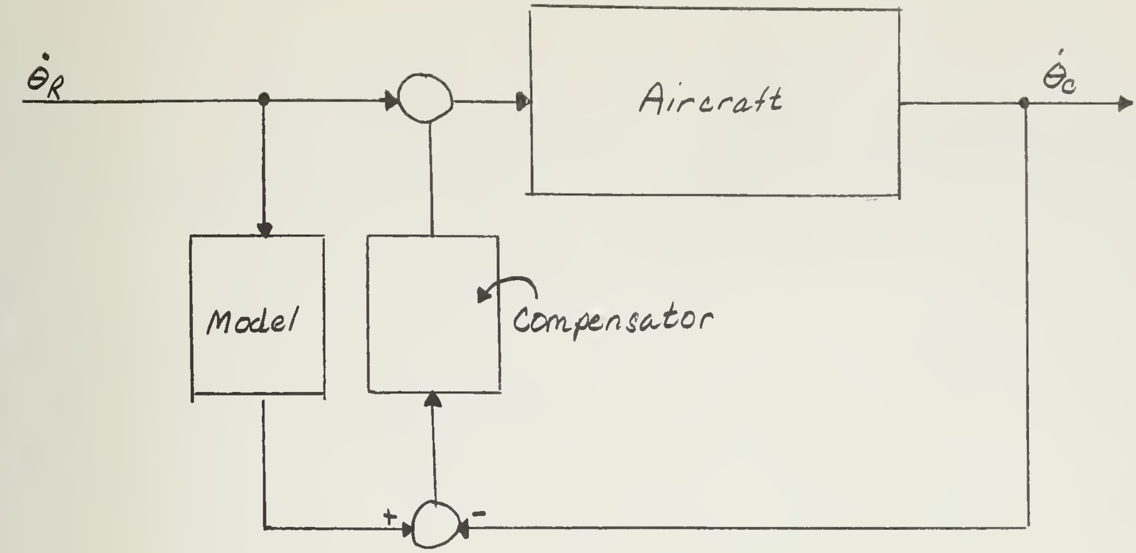


Fig. 9. Conditional Feedback Applied to an Aircraft Problem.

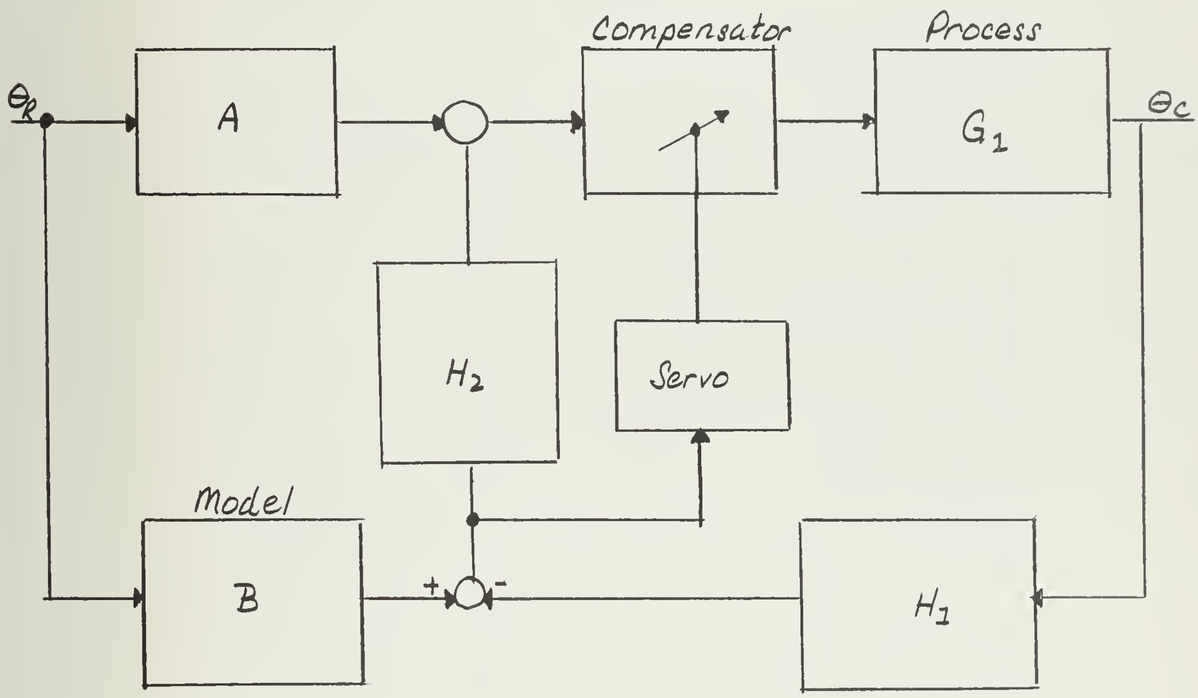


Fig. 10. Conditional Feedback with Variable Parameter Adaptation.

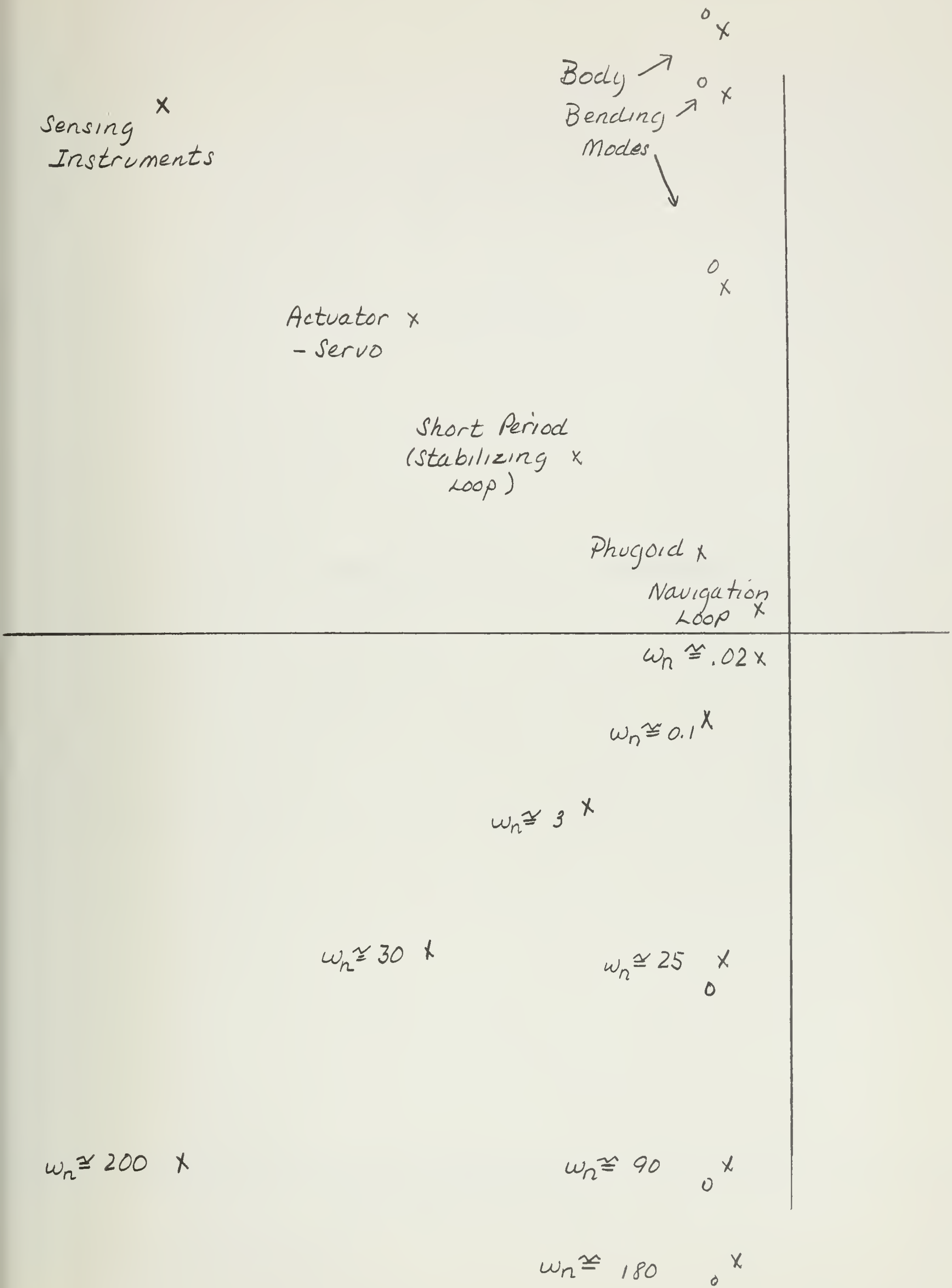


Fig. 11. Roots of the Characteristic Equation of an Automatic Flight Control System.

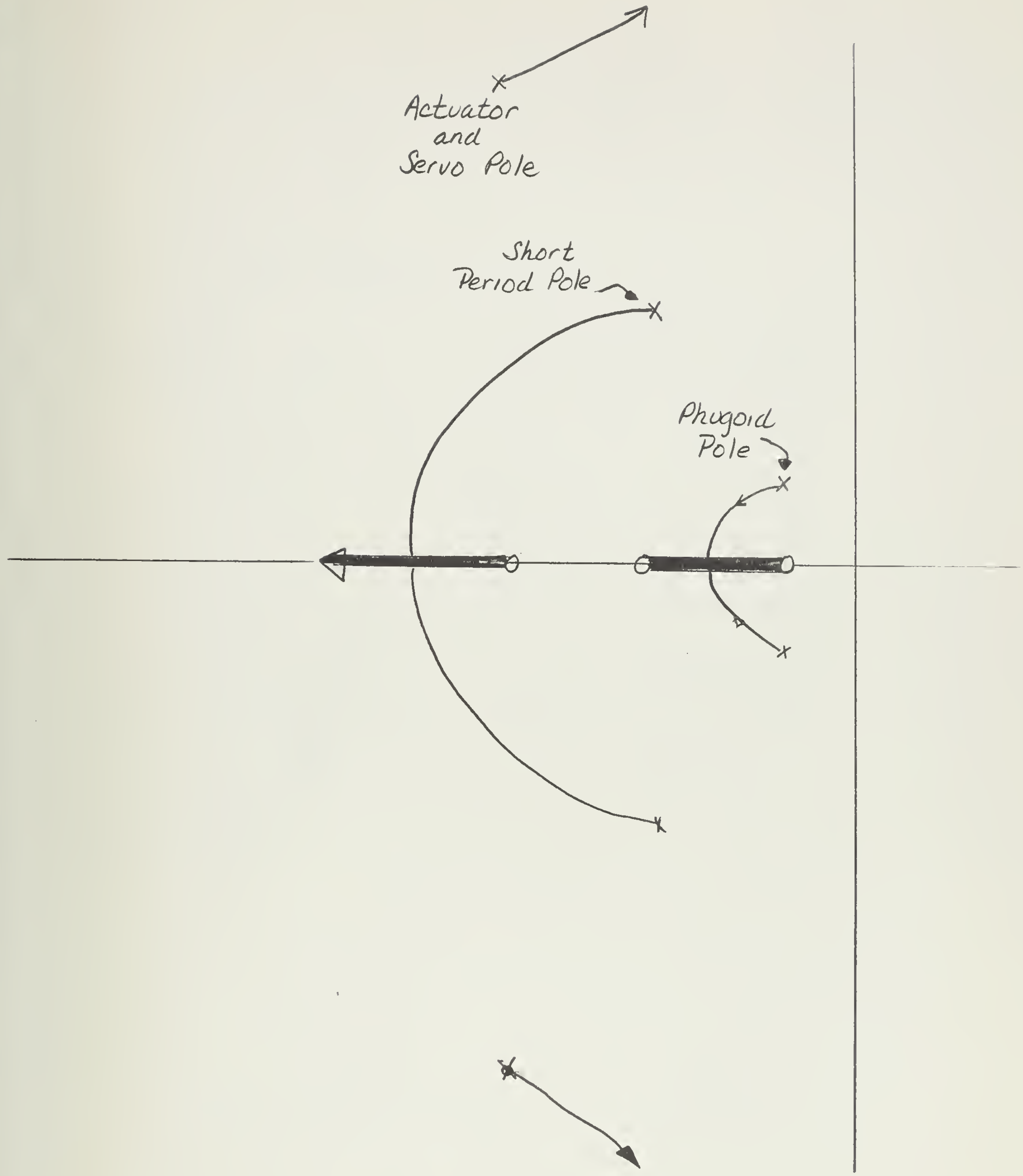


Fig. 12. Root Locus Plot Showing Stabilization to Produce Real Roots in Phugoid and Short Period Modes.

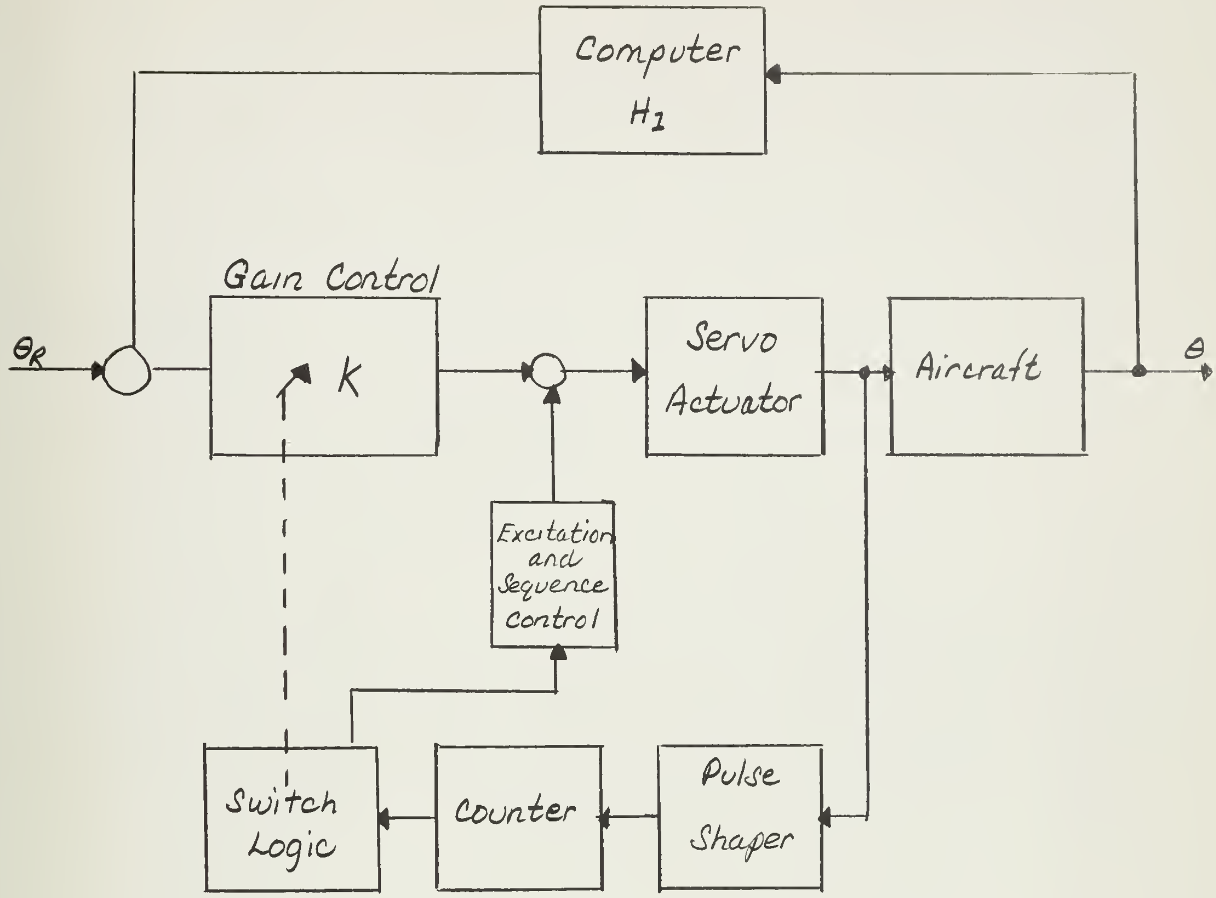


Fig. 13. Mechanization of Gain Adjustment.

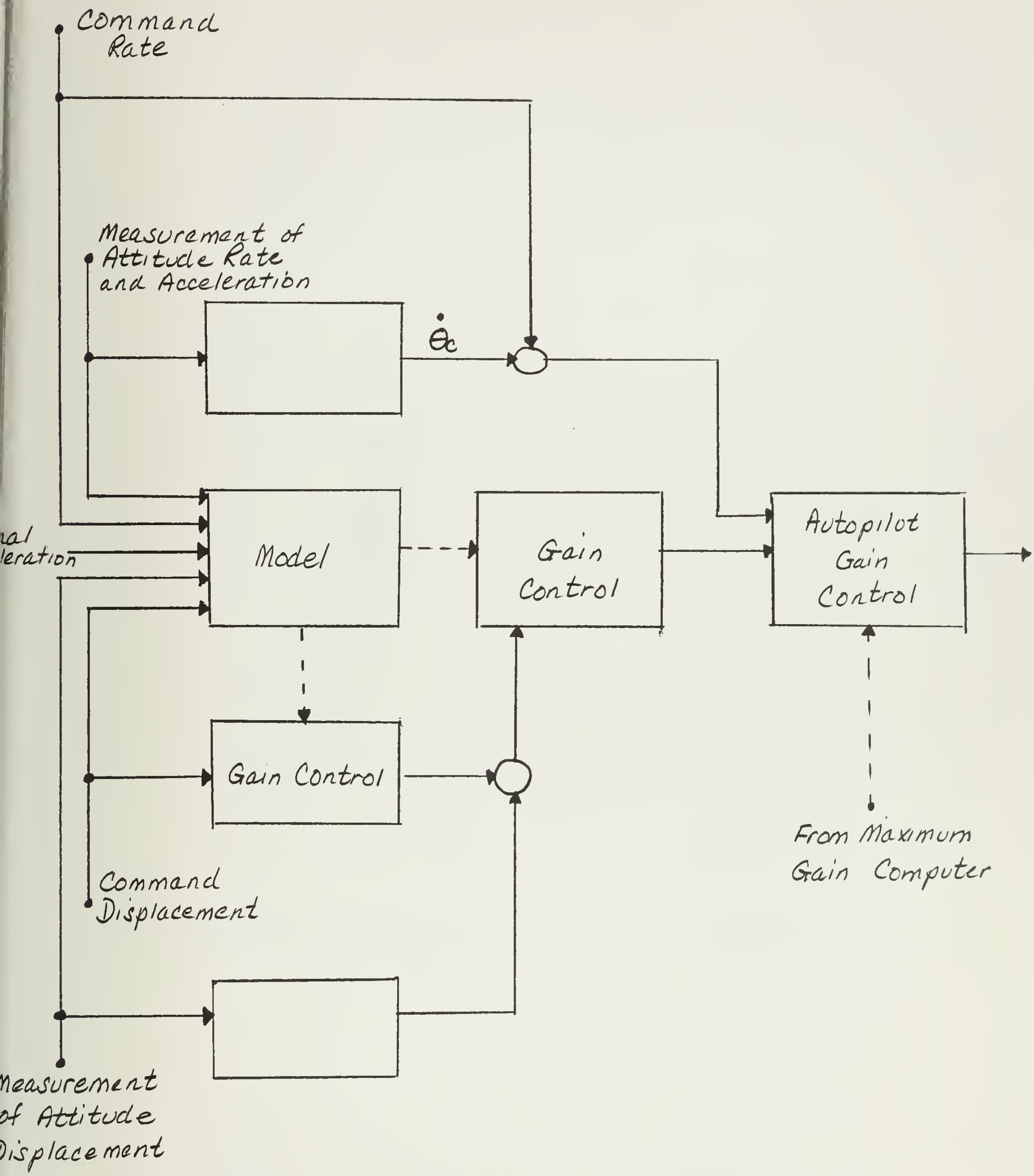


Fig. 14. Block Diagram of Command and Damping Compensation Scheme.

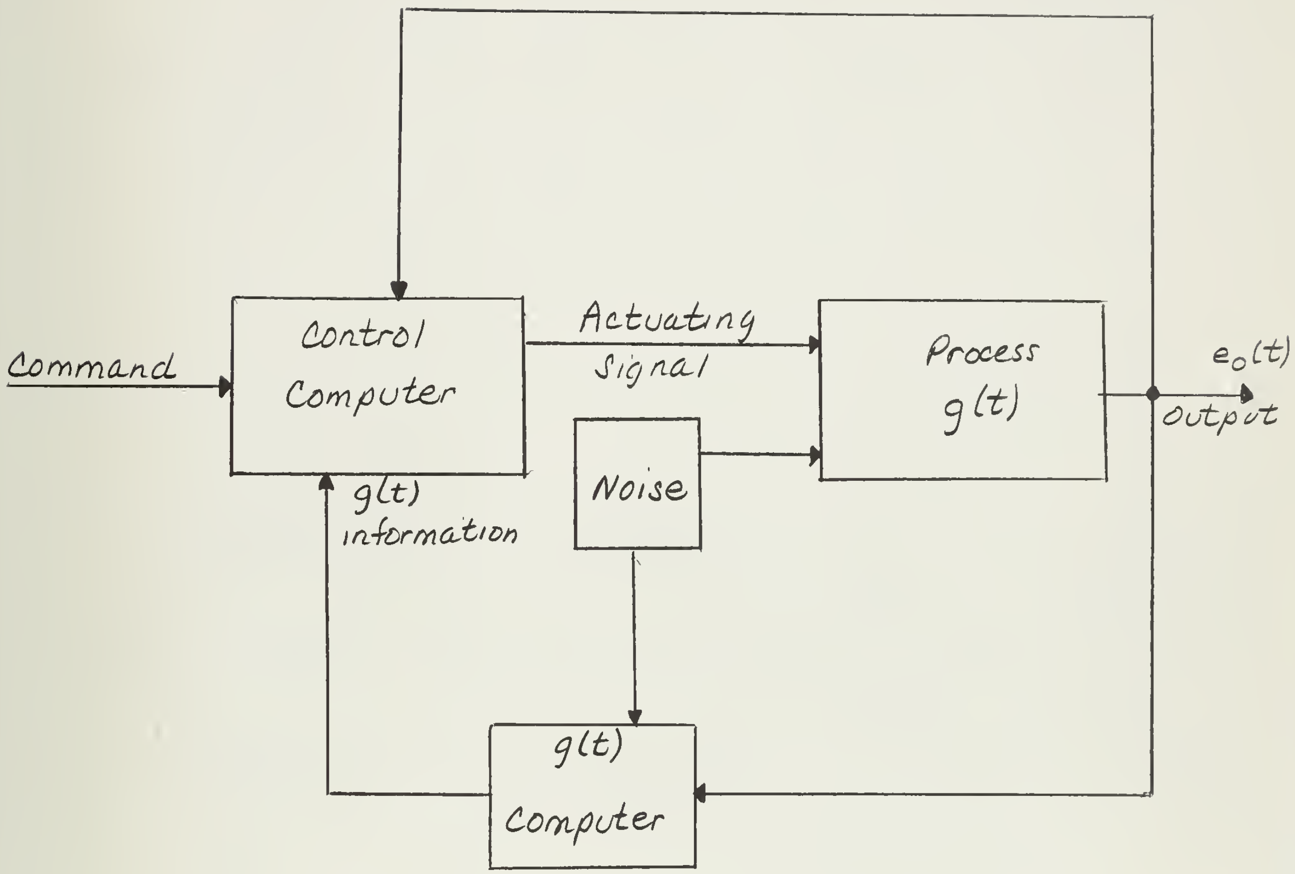


Fig. 15. Adaptive Control Using Impulse Response Measurement.

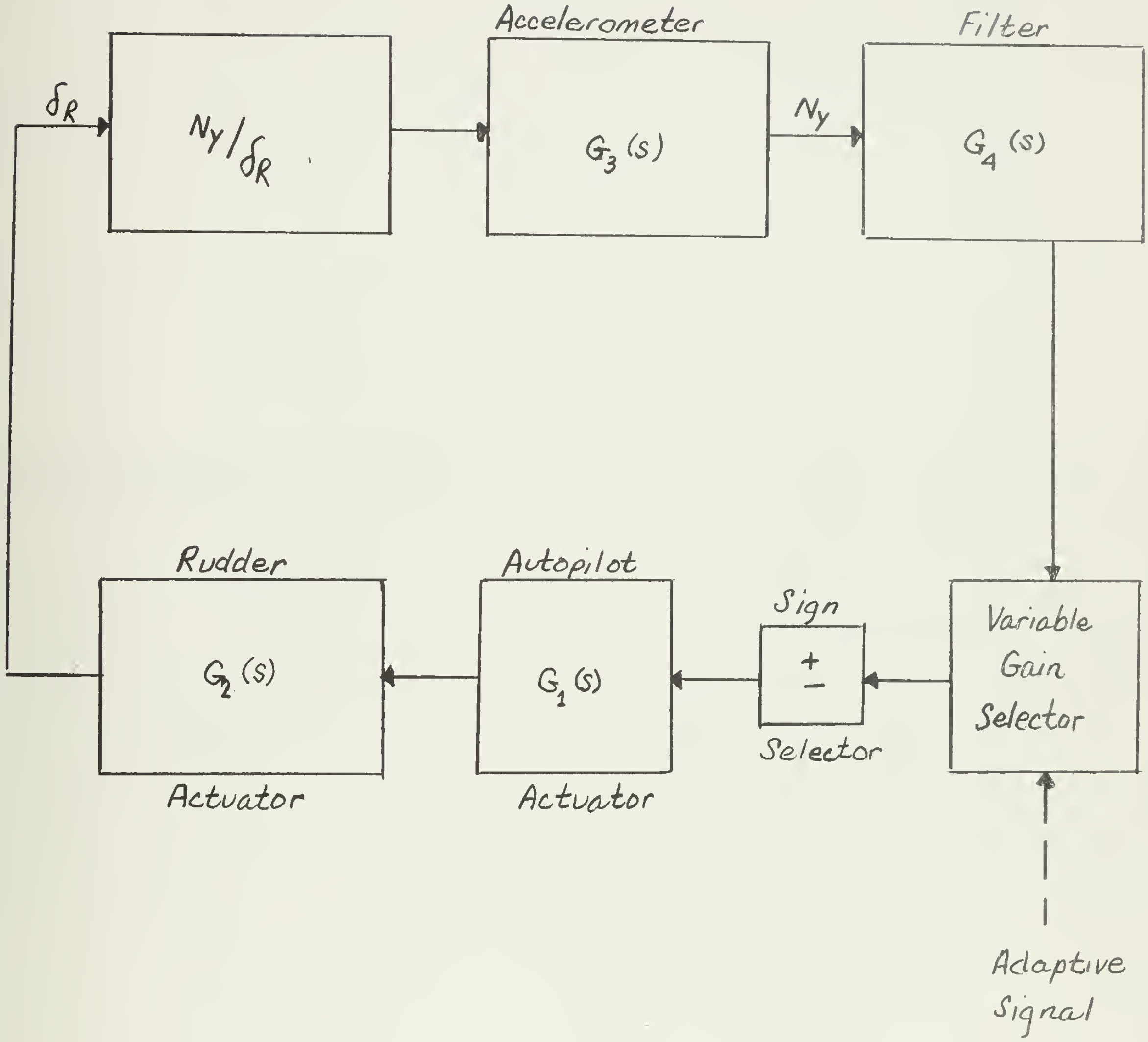
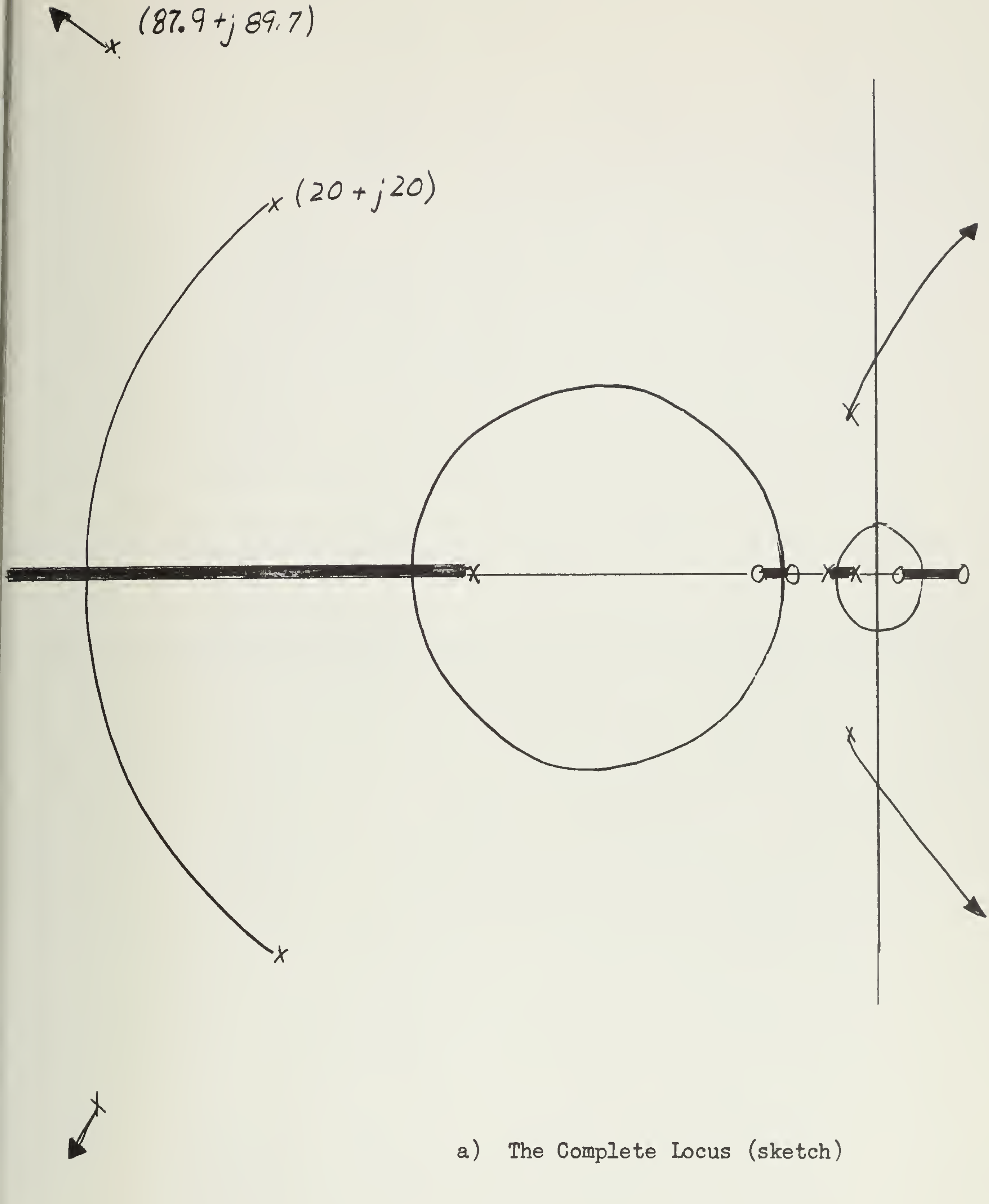
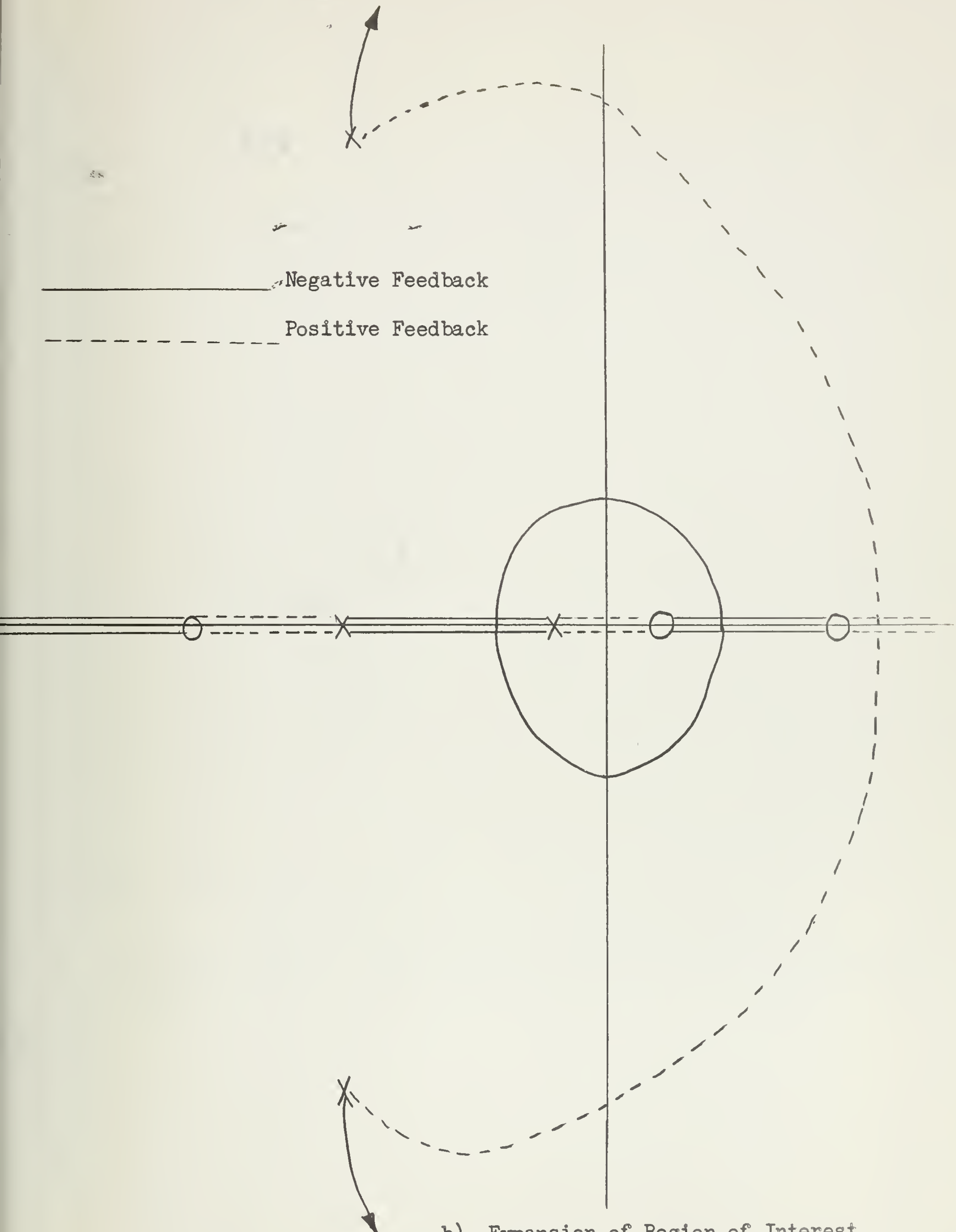


Fig. 16. Block Diagram of Stabilization Loop.



a) The Complete Locus (sketch)

Fig. 17. General Pole-Zero and Root Locus Configuration for Supersonic Aircraft.



b) Expansion of Region of Interest

Fig. 17. General Pole-Zero and Root Locus Configuration for Supersonic Aircraft.

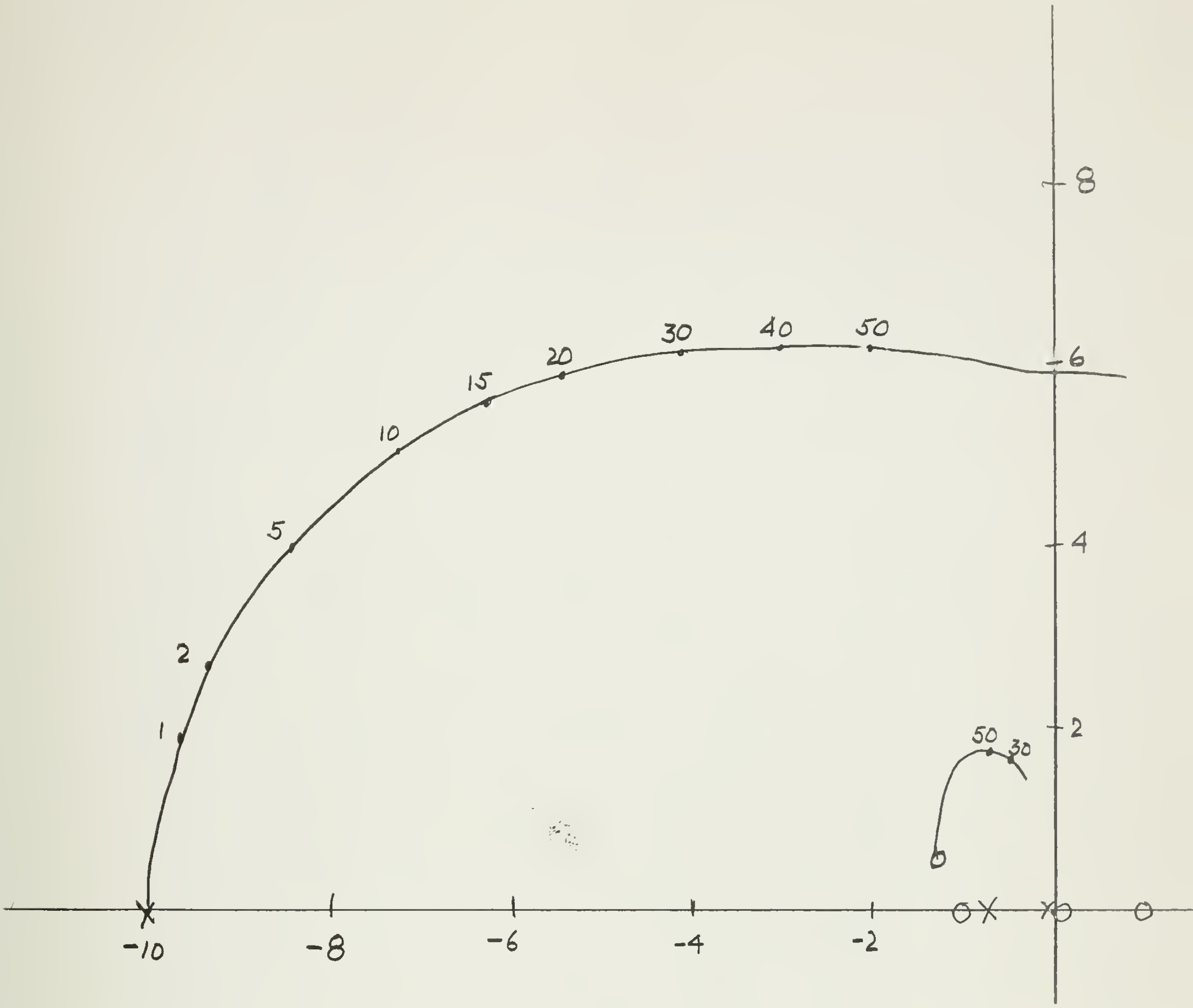


Fig. 18a. Root Locus Plot for

$$\frac{.1869 \times 10^{8} K_{Ny}(s-0.95)(s-0.084)(s+1.32\mp j0.52)(s+1)}{(s+0.066)(s+0.73)(s+0.47\mp j1.58)(s+10)(s+20\mp j20)(s+87.9\mp j89.7)(0.1s+1)}$$

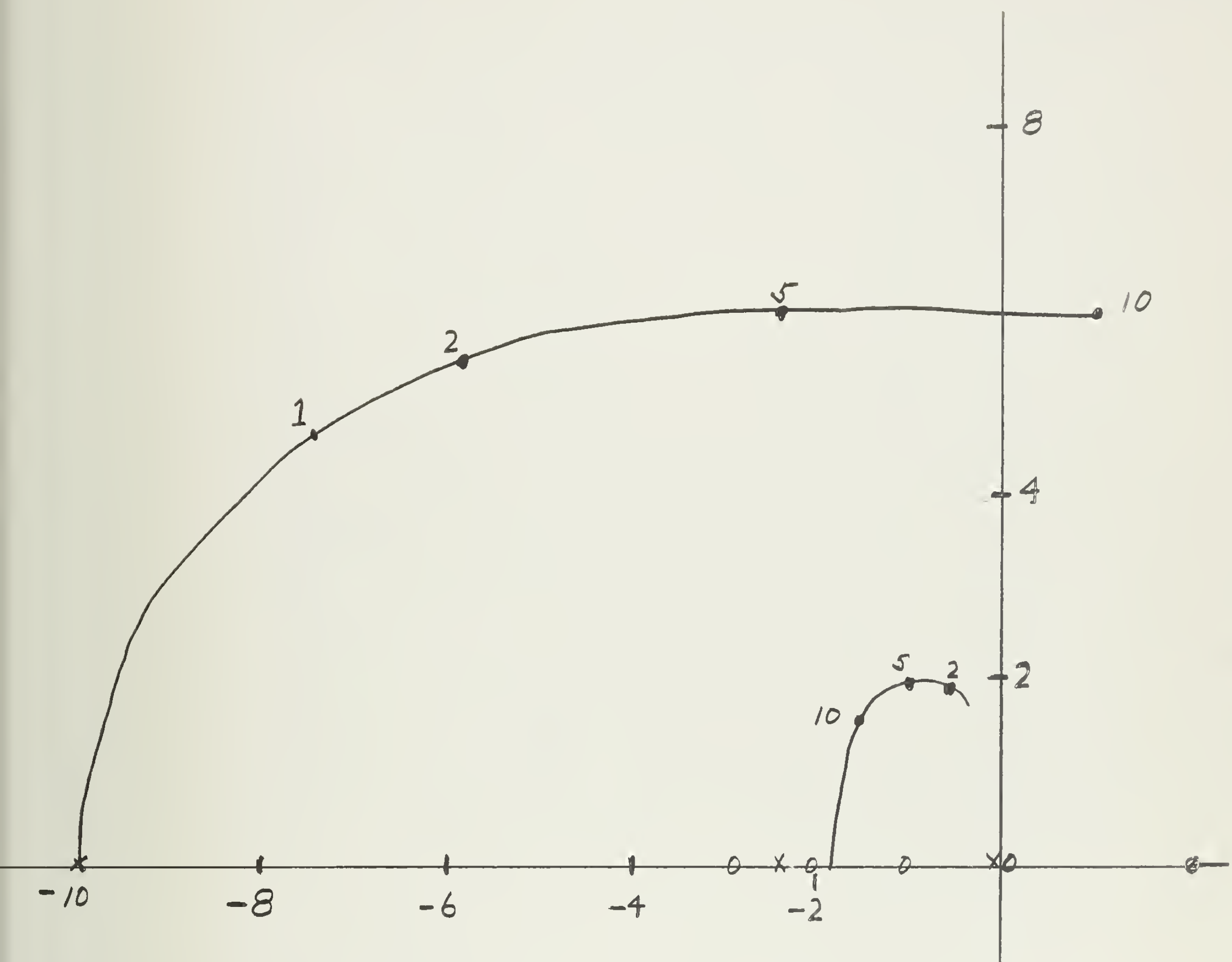


Fig. 18b. Root Locus Plot for

$$\frac{0.6716 \times 10^{8} K_{Ny}(s-0.018)(s+2.94)(s-2.12)(s+2.1)(s+1)}{(s+0.019)(s+2.4)(s+0.34\mp j1.72)(S+10)(s+20\mp j20)(s+87.9\mp j89.7)(0.1s+1)}$$

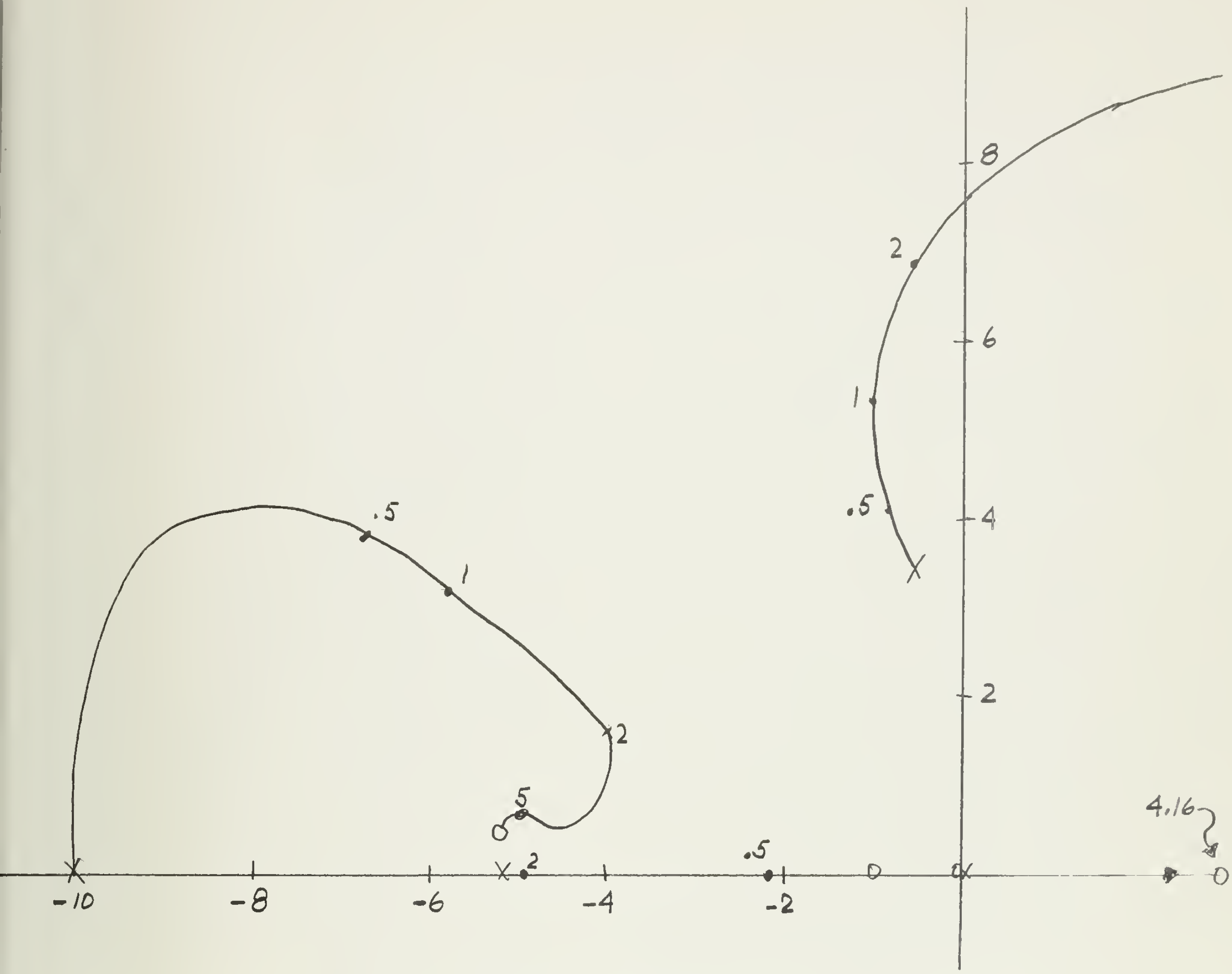


Fig. 18c. Root Locus Plot for

$$\frac{4.2943 \times 10^{8} K_{Ny}(s+0.006)(s-4.17)(s+5.22 \mp j0.5)(s+1)}{(s-.0007)(s+5.21)(s+.053 \mp j3.49)(s+10)(s+20 \mp j20)(s+87.9 \mp j89.7)(0.1s+1)}$$

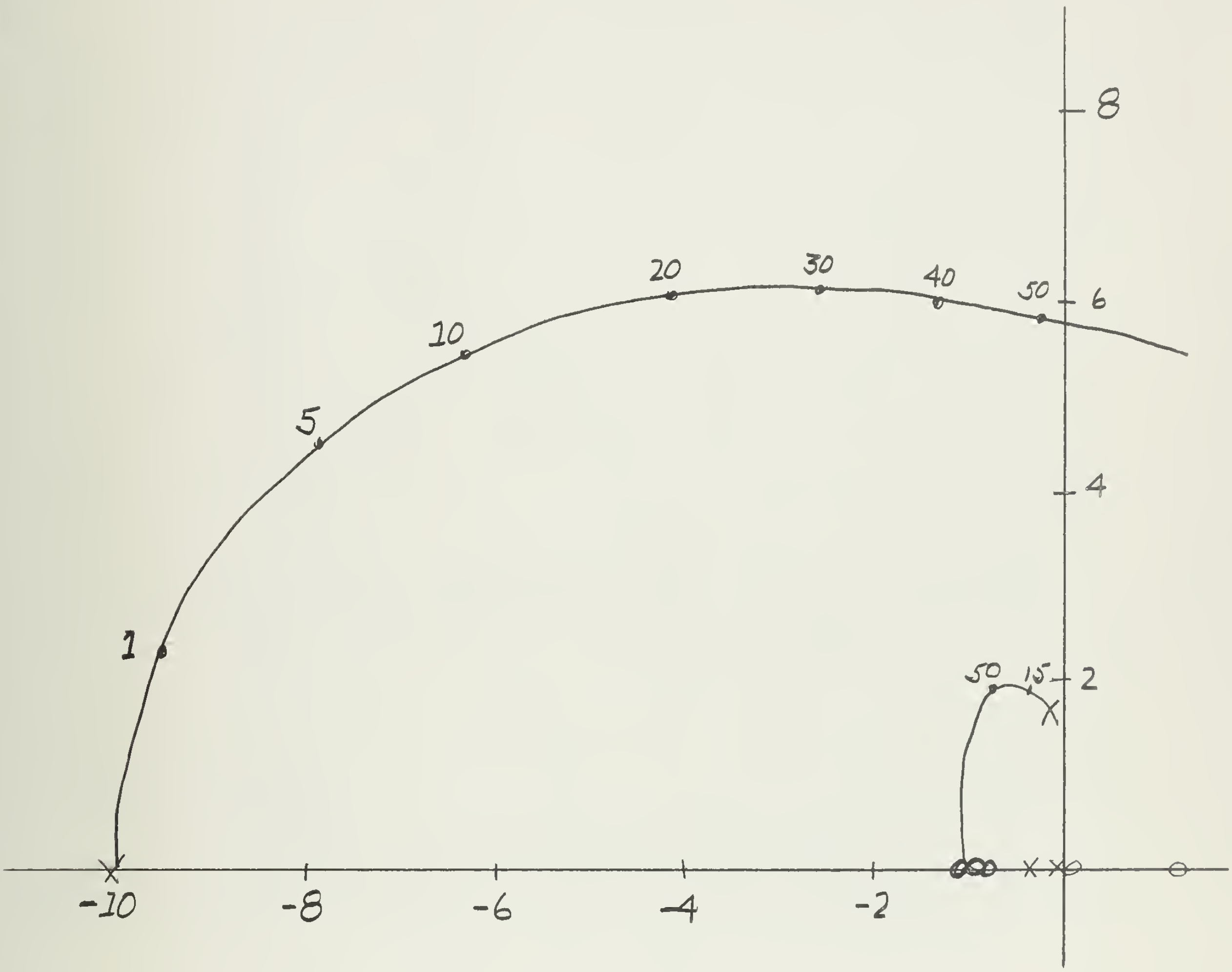
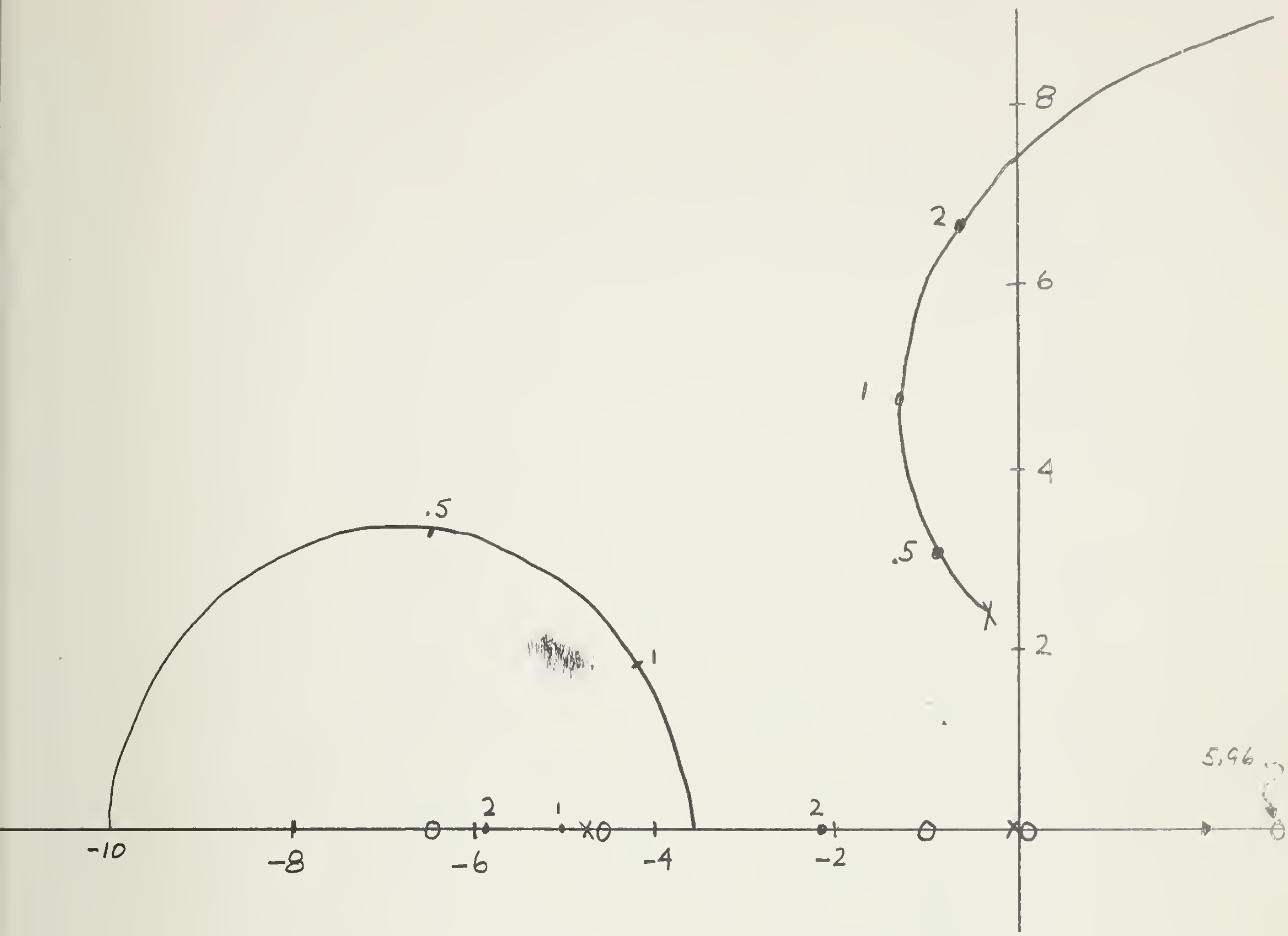


Fig. 18d. Root Locus Plot for

$$\frac{0.276 \times 10^{8} K_{Ny}(s-0.013)(s-1.22)(s+1.06)(s+1)}{(s+0.019)(s+0.33)(s+019\mp j1.67)(s+10)(s+20\mp j20)(s+87.9\mp j89.7)(0.1s+1)}$$



. 18e. Root Locus Plot for

$$\frac{4.1833 \times 10^{8} K_{Ny}(s-0.007)(s+4.69)(s-5.96)(s+6.49)(s+1)}{(s+0.04)(s+4.78)(s+0.36\mp j2.38)(s+10)(s+20\mp j20)(s+87.9\mp j89.7)(0.1s+1)}$$

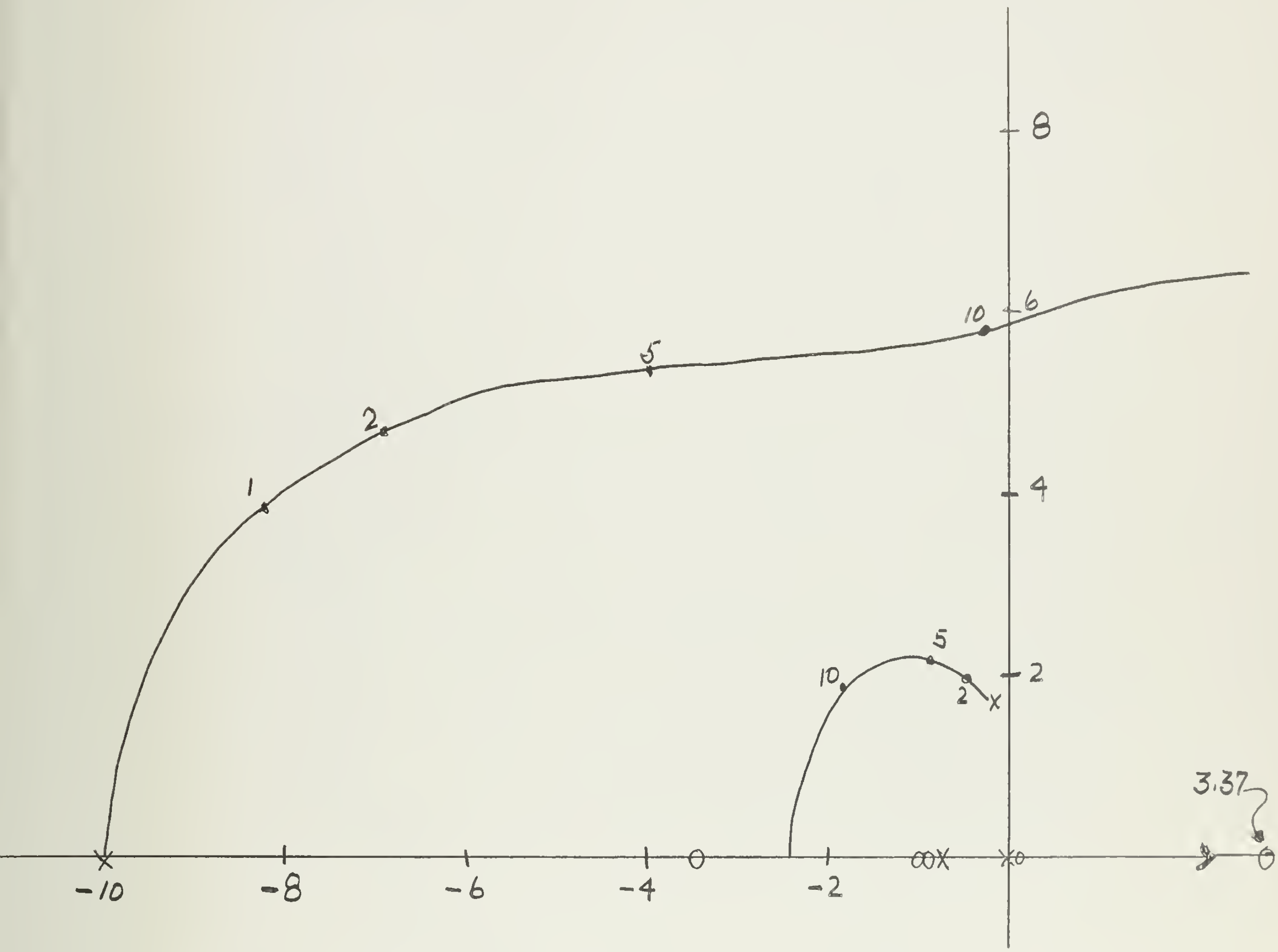


Fig. 18f. Root Locus Plot for

$$\frac{1.062 \times 10^{8} K_{Ny}(s-.001)(s+0.92)(s-3.38)(s+3.49)(s+1)}{(s+0.008)(s+0.74)(s+0.17\mp j1.71)(s+10)(s+20\mp J20)(s+87.9\mp j89.7)(0.1s+1)}$$

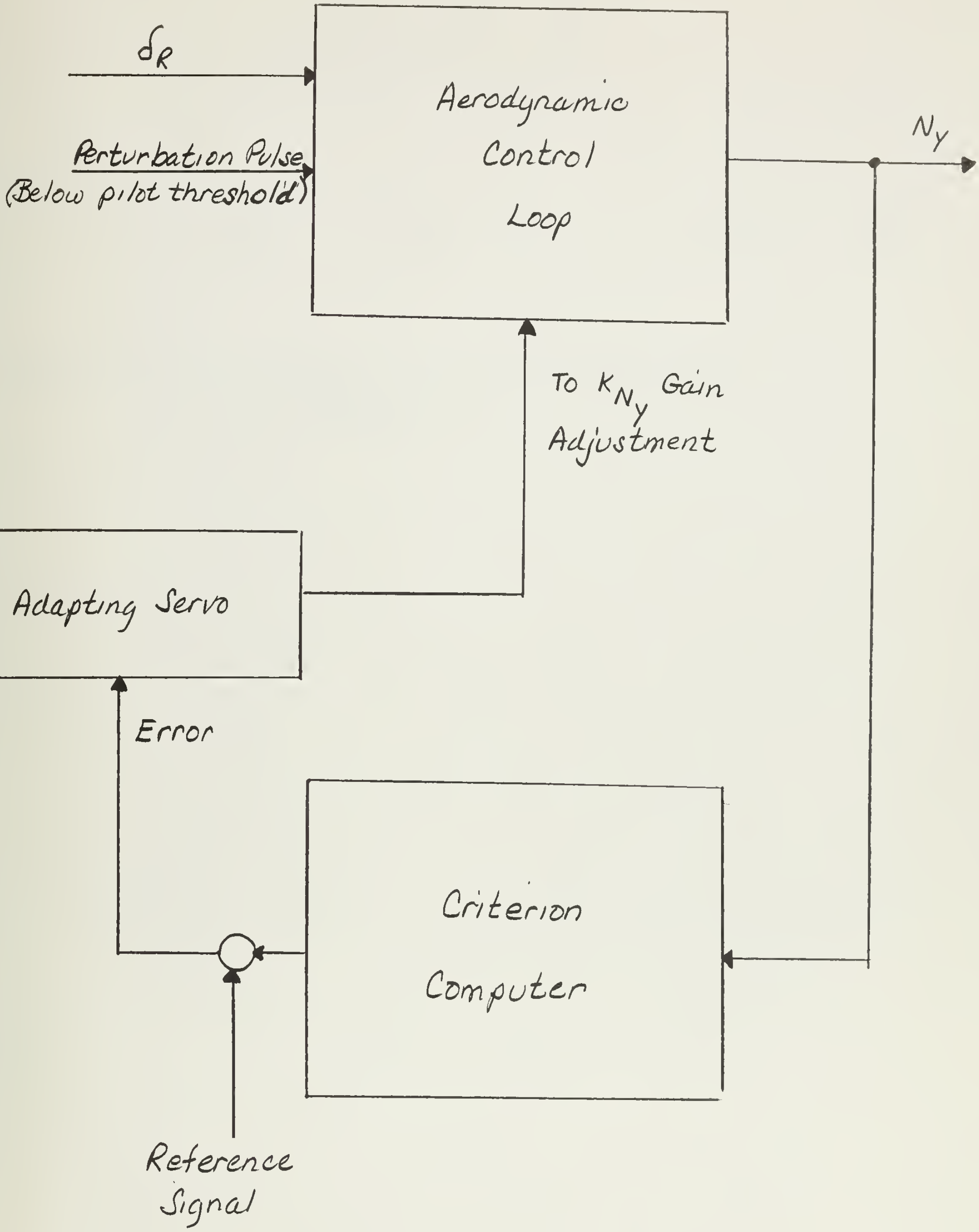


Fig. 19. A Possible Adaptive Scheme.

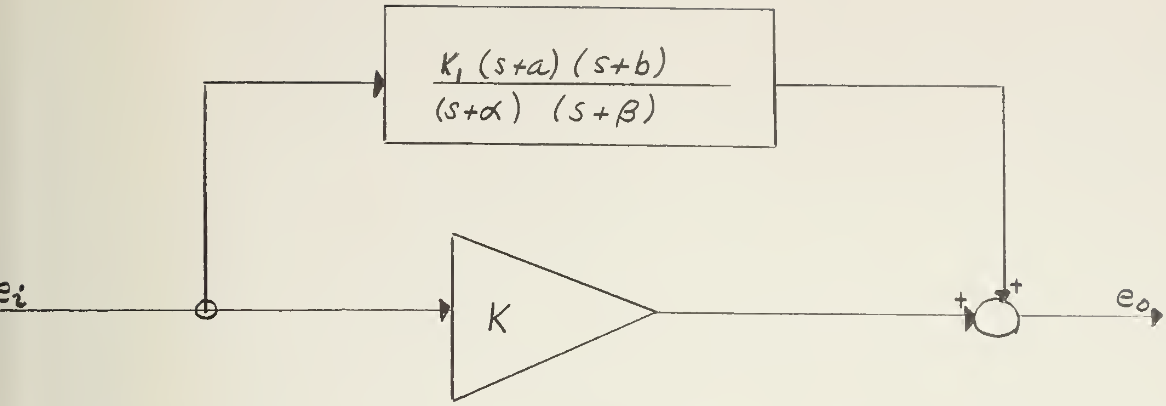


Fig. 20. An Active Network Scheme for the Generation of a Pair of Complex Zeros.

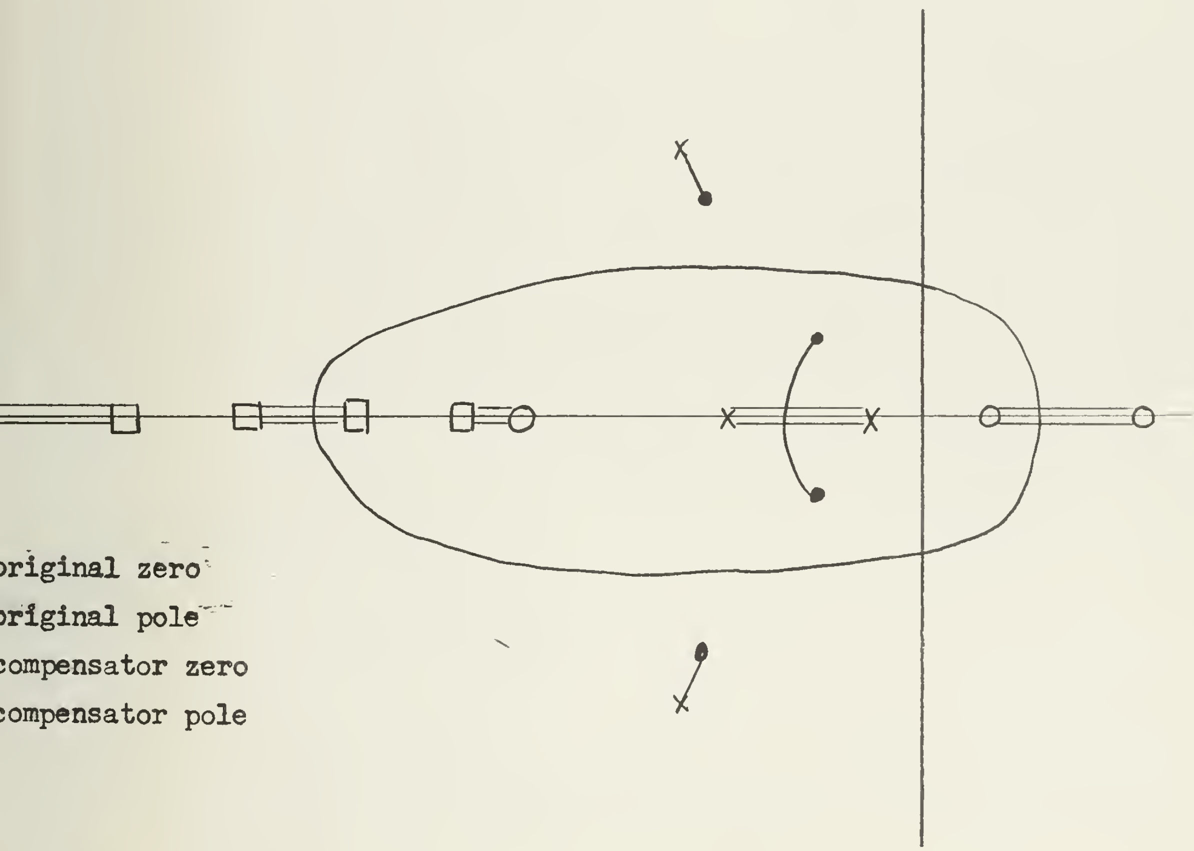


Fig. 21. Possible Root Locus for Compensated System (Negative Feedback).

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